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KEY

TO

RAY'S ARITHMETIC,

PART THIRD:

AND

TO SOME OF THE MORE DIFFICULT QUESTIONS IN

PART SECOND.

NEW AND IMPROVED EDITION.

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ECLECTIC EDUCATIONAL SERIES.

RAY'S
ARITHMETICAL KEY:

CONTAINING

SOLUTIONS TO THE QUESTIONS

IN

RAY'S THIRD BOOK,

AND TO SOME OF THE MOST DIFFICULT QUESTIONS IN

THE SECOND BOOK:

By JOSEPH RAY, M. D.,
PROFESSOR OF MATHEMATICS IN WOODWARD COLLEGE.

NEW AND IMPROVED EDITION.

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PREFACE.

The leading objects in the preparation of the following work, have been:—

First, to aid the teacher, who may not at all times have leisure, in the midst of his labors in the school-room, to furnish the pupil at once with the aid which he requires. For this purpose, the work contains full solutions to all the questions in Third Book, as far as Compound Numbers; after that, the method of working each example is indicated by arithmetical signs, so that any one acquainted with the elementary rules, can readily perform the required operations.

Second, to assist young teachers. For this purpose, besides what has been already named, the work contains solutions to all the difficult questions in the author's Mental Arithmetic, (Second Book); remarks designed to illustrate more fully the nature and principles of several parts of arithmetic; with suggestions on the best methods of instruction.

It has been a complaint with some respectable teachers, that answers are given to the examples in the Third Book. When the pupil is required to explain, at the blackboard, the method of working out an example, *it is desirable that he should rely on the correctness of his own reasoning*, and, *if he understands the method* of obtaining the true result, it is of little moment whether he has seen the answer or not.

The experience of many eminent teachers is, that the use of a book without answers has an immoral effect on the pupils.[*] A superior pupil will obtain the answers correctly, while other members of the class will procure them privately, either from him or others, who have previously worked them.

It is the author's firm conviction, founded on an experience of many years, that the only proper test of a learner's knowledge is, to place him at the blackboard, to solve and explain the questions; and, with this test, it appears to him that the objection to furnishing answers to the questions in the book is removed.

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[*] **A celebrated Boston teacher once said, that an Arithmetic without answers, tended to make the pupils both rogues and liars.**

2559003

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RAY'S HIGHER ARITHMETIC, embracing the principles of the science, analyzed and applied; designed for advanced students and business men.

An admirable and highly commended treatise, peculiar for its freshness, philosophical accuracy and adaptation to the purposes of instruction.

Its rapid introduction into schools of the highest order, and the continued preference given to it, over other admired and popular Arithmetics, may be regarded as evidence of decidedly superior excellence.

OBSERVE.—That it is especially adapted to those desiring enlarged intelligence with reference to Mercantile, Mechanical and Agricultural pursuits.

Just Published.—A KEY to Ray's Higher Arithmetic, embracing full and lucid solutions to the examples contained in that work.

KEY

TO

RAY'S ARITHMETIC,

THIRD BOOK.

SUGGESTIONS ON TEACHING WRITTEN ARITHMETIC.

In teaching written, or, as it is frequently termed, practical arithmetic, the first step is to make the learner well acquainted with numeration and notation. After this, pupils should be frequently exercised in reading and writing numbers.

Similar remarks apply to the numeration and notation of decimals; when these have been properly studied, the subject of decimal fractions presents few difficulties to the student.

It is scarcely necessary to mention that every pupil should be able to perform Addition, Subtraction, Multiplication, and Division, with *ease* and *correctness*, before proceeding further. In order to do this, it is frequently necessary to review these subjects more than once, and to require the pupil to solve examples additional to those contained in the text-book. The instructor and pupil should both feel that no time is lost in dwelling upon and reviewing the elementary rules, until the pupil is thoroughly master of them; it is the most certain means of insuring his future rapid and successful progress.

In explaining the nature and operation of many of the rules, the instructor will find it of great advantage to illustrate the subject by simple mental exercises. Every competent teacher can generally frame such as are suitable, and present them orally to the class. In this way, they make a more decided impression on the pupils, than when presented in the text-book.

Mental exercises, suitable for the illustration of those subjects, when they can be used with advantage, will be found in a succeeding part of this volume, under their appropriate heads, and preceding the examples for exercise at the black-board.

When practicable, the pupils should be arranged in classes, due regard being had to their ages, acquirements, etc. After this, the proceeding in the best schools, is somewhat as follows:

A certain number of examples is arranged for a lesson; it will, also, frequently be necessary that a part, or even the whole, of the lesson shall consist of the illustration of principles, or the memorizing of definitions or rules. When the class meets for recitation, each pupil passes his slate into the hands of the pupil next above him, except the pupil at the head, who passes his to the foot scholar. The teacher then reads the answer to the first question, while each pupil examines the slate he holds, to see if the answer is correct and properly obtained.

In addition to reading the answer, the teacher, in many cases, such, for example, as proportion, should state the general method of working the question. The pupils mark the answers that are wrong, or obtained improperly. In the same manner, each question is examined and marked. Instead of the teacher reading the answers, the pupils in succession may read them.

When there is a black-board (and there should be at least one in every school-room, 4 or 5 feet wide, and as long as the room will permit), each pupil should be required to work out one or more of the examples, and give the reasons for the method of performing the operation. The time required to examine the questions is generally short, while the habit of closely scrutinizing each other's work, improves the perceptive faculties of the pupils.

The instructor will find the suggestion contained in the Note, Article 138, with reference to the examples having a star, *, worthy of particular attention.

In order to teach either intellectual or practical arithmetic well, the instructor must be familiar with the subject, so as to be able to explain, promptly and clearly, the solution of every question, as well as the principles on which each rule is based. He should have, also, a strong desire to render his pupils familiar with the subject: for, it may be stated as an acknowledged truth in education, that, a teacher who is indifferent to the advancement of his pupils, will render them indifferent likewise; and that, on the contrary, one that is deeply interested in the student's progress, will rarely fail to enkindle the same spirit.

OF SYMBOLS.

1. The sign $=$ is called the sign of *equality;* it denotes that quantities between which it is placed are equal. Thus, $\$1=100$ cents; read, one dollar equals one hundred cents.

2. The sign $+$ is called *plus;* it denotes that numbers between which it is placed are to be added together. Thus, $4+3=7$; read, four *plus* three equals seven.

3. The sign $-$ is called *minus;* it denotes that the number placed at the right of it is to be taken from the number on the left. Thus, $5-3=2$; read, five *minus* three equals two.

4. The sign $\times$ is called the sign of *multiplication;* it denotes that the two quantities between which it is placed are to be multiplied together. Thus, $4\times3=12$; read, four times three equals twelve.

5. The sign $\div$ is called the sign of *division;* it denotes that the number on the left of it is to be divided by that on the right. Thus, $12\div3=4$; read, twelve divided by three is equal to four.

Division is also denoted thus, $\frac{12}{3}$, or $3)12$; either of which indicates that twelve is to be divided by three.

6. The *parenthesis,* ( ), when it incloses several numbers, indicates that these are to be regarded as one single number. Thus, $(5+3)\times4=32$; read, five plus three multiplied by four is equal to thirty-two.

If the parenthesis were wanting, the expression would stand thus, $5+3\times4=17$; this indicates that five is to be added to the product of three multiplied by four, and that the sum is seventeen.

7. A number placed over another number, a little to the right, is called an *exponent.* Thus, 2 and 3 are called the exponents, in the expressions, 4^2, 5^2, etc., 4^3, 5^3, etc.

The exponent of a quantity denotes that the quantity is to be used as a factor as many times as there are units in the exponent. Thus, $4^2=4\times4$, $2^3=2\times2\times2$, etc.

The exponent 2 denotes the *second* power, or square of the number over which it is placed. The exponent 3 denotes the *third* power, or cube. Thus, $3^2=3\times3=9=$ the square of three; and $3^3=2\times2\times2=8=$the cube of two.

8. The sign $\sqrt{}$ denotes that the *square* or *second* root of the number before which it is placed, is to be extracted. Thus, $\sqrt{16}=4$; read, the square root of sixteen is equal to four.

In a similar manner the sign $\sqrt[3]{}$ denotes that the *cube* or *third* root of the quantity before which it is placed is to be extracted. Thus, $\sqrt[3]{27}=3$; this is read, the cube root of twenty-seven is equal to three.

NUMERATION.

SUGGESTIONS TO TEACHERS.—Be careful to impress the mind of the pupil with a clear idea of a *unit*. Show him that numbers are formed by the successive addition of units. Explain the distinction between units of different orders; this may be done conveniently by a reference to the coins of the United States, stiff pieces of paper being used to represent units of the first order; those of the second order will be represented by cents; those of the third, by dimes; and those of the fourth, by dollars. Make the pupils well acquainted with *reading* numbers under one thousand before proceeding higher. Until the pupil is familiar with the subject, require him to point the numbers into periods of three places each, before attempting to read them. Let the pupil be made well acquainted with the method of writing numbers under 1000; then, by the use of points, he may soon be taught to write any number with ease and accuracy.

TO TEACHERS.—Where the word "ARTICLE" occurs in this volume, it has reference to the corresponding "ART." in "Ray's Third Book."

The examples in the Key are numbered, and refer to the corresponding number of example, under the same article, in the Arithmetic.

ARTICLE 13.

1, Five. 2, Sixty-three.
3, Ninety. 4, One hundred.
5, One hundred and four.
6, One hundred and forty-seven.
7, Two hundred and eight.

8, Two hundred and eighty.

9, Four hundred and three.

10, Seven hundred and twenty-nine.

11, Seven hundred and ten.

12, Nine hundred and one.

13, One thousand.

14, One thousand and five.

15, One thousand and fifty.

16, One thousand and eighty-five.

17, One thousand one hundred.

18, One thousand one hundred and eight.

19, Three thousand and three.

20, Four thousand and fifty.

21, Three thousand and forty-five.

22, Nine thousand six hundred and ninety-nine.

23, Ten thousand.

24, Twelve thousand.

25, Thirteen thousand two hundred.

26, Fifty thousand and four.

27, Sixty-two thousand and one.

28, Seventy thousand four hundred.

29, Eighty thousand and ninety.

30, Ninety-seven thousand and ten.

31, Forty thousand three hundred and five.

32, Seventy-six thousand and fifty-two.

33, Eighty-three thousand nine hundred and ninety-one.

34, Two hundred thousand.

35, Four hundred and fifty-six thousand.

36, Six hundred and eighty-two thousand three hundred.

37, Seven hundred and four thousand two hundred and eight.

38, Eight hundred thousand one hundred and forty-one.

39, Nine hundred thousand and sixteen.

40, Six hundred and one thousand and twenty.

41, Seven hundred thousand four hundred.

42, Eight hundred thousand and two.

43, Nine hundred and ten thousand one hundred and three.

44, Seven hundred thousand one hundred.

45, Twenty thousand and twenty.

46, One hundred and six thousand three hundred and seven.

47, Four hundred thousand and one.

48, Three hundred and two thousand four hundred and four.

49, Eight hundred thousand and ten.

50, Seven hundred thousand and ten.

51, One million, thirty thousand seven hundred and twenty-five.

52, Four millions, fifty thousand six hundred and seven.

53, Six millions, six hundred and one thousand.

54, Seven millions, four hundred and six thousand and thirty-five.

55, Nine millions, seven hundred and twenty-five thousand and fourteen.

56, Three hundred thousand three hundred.

57, Nine hundred and nine thousand and ninety.

58, Eight millions, six hundred thousand and fifty.

59, Two millions, one hundred and two thousand one hundred and two.

60, Four millions, eighty thousand four hundred.

61, One million and eleven.

62, Three millions, thirty-six thousand.

63, Nine millions, one thousand and three.

64, Thirty-three millions, seven thousand eight hundred and twenty.

65, Sixty-one millions, one hundred and eighty-nine thousand six hundred and two.

66, Ninety-nine millions, ninety-nine thousand and ninety-nine.

67, One hundred and thirty millions, six hundred and seventy thousand nine hundred and twenty-one.

68, Six billions, nine hundred millions, seven hundred and two thousand and nine.

69, Twenty-three billions, four millions, ninety thousand nine hundred and one.

70, Nine hundred and forty-two billions, twenty-nine millions, three hundred and seven thousand and twenty-nine.

Article 14.

1. 100. One hundred.
2. 203. Two hundred and three.
3. 3020. Three thousand and twenty.
4. 50204. Fifty thousand two hundred and four.
5. 9000500. Nine million, five hundred.
6. 70050. Seventy thousand and fifty.
7. 600004000. Six hundred million, and four thousand.
8. 20050200. Twenty millions, fifty thousand two hundred.
9. 5 in the first order is five, or five units. 4 in the second order is forty. 3 in the sixth order is three hundred thousand. 8 in the third order is eight hundred. 7 in the fourth order is seven thousand. 9 in the seventh order is nine millions. 4 in the fifth order is forty thousand. 2 in the eighth order is twenty millions. 8 in the 6th order is eight hundred thousand. 4 in the 9th order is four hundred millions. 5 in the 10th order is five billions.

NOTATION.

ARTICLE 15.

1. 4
2. 16
3. 18
4. 20
5. 24
6. 28
7. 30
8. 32
9. 37
10. 41
11. 46
12. 59
13. 65
14. 97
15. 87
16. 104
17. 120
18. 130
19. 175
20. 203
21. 340
22. 435
23. 502
24. 625
25. 222
26. 999
27. 807
28. 870
29. 901
30. 606
31. 309
32. 190
33. 202
34. 101
35. 21
36. 300
37. 503
38. 827
39. 1000
40. 1020
41. 25006
42. 345
43. 760
44. 3406
45. 42030
46. 30000
47. 163000
48. 341563
49. 45083026
50. 870743579
51. 2804
52. 4029
53. 6006
54. 22765
55. 80201
56. 90001
57. 30030
58. 410205
59. 800669
60. 900001
61. 500050
62. 100010
63. 909090000
64. 100010001
65. 91007060
66. 70000004
67. 700010001
68. 1001000040
69. 40000200005
70. 726050001243.

ADDITION.

SUGGESTIONS TO TEACHERS.—Simple Addition is the method of adding numbers of the *same* name or kind into one sum. We can not add numbers of *different* kinds together. If a reason is asked for this, the reply is, that it is *self-evident;* thus, 2 apples and 3 apples added, make five apples; but we can not add 2 apples and 3 pears together; the sum would be neither 5 apples nor 5 pears. If it is asked why we write units under units, tens under tens, etc., the answer is, since we can only add numbers of the same name together, we write those of the same denomination under each other, merely for the sake of convenience in performing the work. We carry one for every ten, because ten units in any place make a unit of the next higher place or order. If the base of our scale of Notation had been twelve, instead of ten, we should have carried one for every twelve. After the pupil has become acquainted with Compound Addition, he will better understand the reason for carrying in Simple Addition. Clerks, who add long columns of figures with great rapidity, never call the names of the figures in adding; they *look* at each figure as they proceed, and either mentally or orally name the sum which it makes in connection with those that precede it. In this manner the addition is performed in about half the time that it could have been, if each figure had been named aloud.

ARTICLE 20.

(Ex. **3**) sheep.
43
21
14
Ans. 78

(Ex. **4**) dollars.
210
142
35
Ans. 387

(Ex. **5**) 4321
1254
3120
Ans. 8695

(Ex. **6**) 50230
3105
423
Ans. 53758

ARTICLE 22.

(Ex. **11**) *Ans.* 21023.
(Ex. **12**) *Ans.* 27910.
(Ex. **13**) *Ans.* 89569.
(Ex. **14**) *Ans.* 2499593.
(Ex. **15**) *Ans.* 24194086.

(Ex. **16**) 23
41
74
83
16
Ans. 237

(Ex. **17**) 45
19
32
74
55
Ans. 225

(Ex. **18**) 51
48
76
85
4
Ans. 264

(Ex. **19**) 263
104
321
155
Ans. 843

(Ex. **20**) 94753
2847
93688
9386
258
3456
Ans. 204388

(Ex. **21**) Jan. 31
Feb. 28
March 31
April 30
May 31
Ans. 151

(Ex. **22**) June 30
July 31
Aug. 31
Sept. 30
Oct. 31
Ans. 153

(23)
First five, 151
Next five, 153
November, 30
December, 31
Ans. 365

(24) yds.
50
65
42
89
Ans. 246

(25) dollars.
245
325
187
96
Ans. 853

(Ex. **26**) 1732
67
Ans. 1799

(**27**) 1656
1164
180
1004
Ans. 4004

(**28**)

sheep.	lambs.
65	43
187	105
370	243
416	95
Ans. 1038	486

(**29**)

barrels.	dollars.
30	285
18	144
23	235
34	408
Ans. 105	1072

(**30**) 287
596
841
fourth, 1724
Ans. 3448

(**31**) 337
49
431
607
146
Ans. 1570

(**32**) 3005
42627
105
307004
800791
320600
Ans. 1474132

(**33**) 275432
402030
300005
872026
4002347
Ans. 5851840

(**34**) 880000889
2002002
77436000
206005207
49003
990019919
Ans. 2155513020

SUBTRACTION.

Suggestions to Teachers.—For the same reasons as those already given with reference to simple addition, we can only find the difference between numbers of the same kind. The order of writing units under units, is adopted for convenience. It brings the individual figures, between which the subtractions are to take place, in the most convenient position with reference to each other. It is a matter of indifference which of the numbers is placed uppermost. The method of placing the number to be subtracted under that from which it is to be taken, is generally supposed to be the more convenient.

ARTICLE 25.

(2) sheep.
245
123
Ans. 122

(3) dolls.
875
751
Ans. 124

(4) 734
531
Ans. 203

(5) 8752
3421
Ans. 5331

(6) 529
8
Ans. 521

(7) 79484
25163
Ans. 54321

ARTICLE 27.

(12) 4444444
1234567
Ans. 3209877

(13) 91516171
15161718
Ans. 76354453

(14) 63046571
34992884
Ans. 28053687

(15) 153425178
53845248
Ans. 99579930

(16) 100000000
10001001
Ans. 89998999

(17) cents.
63
17
Ans 46

(18) dollars.
137
65
Ans. 72

(19) feet.
75
37
Ans. 38

(20) years.
1837
1492
Ans. 345

(21) dollars.
1840
475
Ans. 1365

(22) dollars.
10104
7426
Ans. 2678

(23) dollars.
100000
11
Ans. 99989

(24) 20010
19019
Ans. 991

(25) 912010
50082
Ans. 861928

(26) 4000000
4004
Ans. 3995996

(27) 2020930
1009006
Ans. 1011924

MULTIPLICATION.

Suggestions to Teachers.—It is important that the pupil should understand the nature of multiplication, namely, that it is a short method of performing addition, when the numbers to be added together are all equal to each other.

The multiplier shows *how many times* the multiplicand is to be repeated, hence it is always an *abstract number*. The multiplicand, however, may be either an abstract or a concrete number; thus, the abstract number 5, repeated three times, makes 15; and the concrete number, 5 apples, repeated three times, makes 15 apples. It is obvious from this, that the product is always of the same name as the multiplicand.

Some, who have only examined the subject superficially, suppose that two concrete numbers may be multiplied together, as in the following question: What will 3 yards of cloth cost, at 5 dollars a yard? The answer, it is obvious, is 15 dollars. Now, it is asserted, that in this case we multiply 5 dollars by 3 yards, and that the product is 15 dollars. This view arises from an ignorance of analysis and the elementary principles of numbers. The correct solution is as follows: *The price of* 1 *yard is* 5 *dollars, and* 3 *yards being three times as much as* 1 *yard, the price will be three times as much, that is,* 15 *dollars.* (See Ray's Arithmetic, Second Book, page 37.)

Article 31.

(12) 49
3
Ans. 147

(13) 57
4
Ans. 228

(14) 128
5
Ans. 640

(15) 367
6
Ans. 2202

(16) 1427
7
Ans. 9989

(17) 19645
8
Ans. 157160

(18) 44386
9
Ans. 399474

(19) 708324
7
Ans. 4958268

(20) 964578
9
Ans. 8681202

(21) 96432
10
Ans. 964320

(22) 46782 11 *Ans.* 514602	**(23)** 86458 12 *Ans.* 1037496	**(24)** 680323 11 *Ans.* 7483553

ARTICLE 32.

(25) 1236839 12 *Ans.* 14842068	**(29)** 235 13 705 235 *Ans.* 3055	**(30)** 346 19 3114 346 *Ans.* 6574
(31) 425 29 3825 850 *Ans.* 12325	**(32)** 518 34 2072 1554 *Ans.* 17612	**(33)** 279 37 1953 837 *Ans.* 10323
(34) 869 49 7821 3476 *Ans.* 42581	**(35)** 294 57 2058 1470 *Ans.* 16758	**(36)** 429 62 858 2574 *Ans.* 26598
(37) 485 76 2910 3395 *Ans.* 36860	**(38)** 624 85 3120 4992 *Ans.* 53040	**(39)** 976 97 6832 8784 *Ans.* 94672
(40) 342 364 1368 2052 1026 *Ans.* 124488	**(41)** 376 526 2256 752 1880 *Ans.* 197776	**(42)** 476 536 2856 1428 2380 *Ans.* 255136

(43) 2187
215
10935
2187
4374
Ans. 470205

(44) 3489
276
20934
24423
6978
Ans. 962964

(45) 1646
365
8230
9876
4938
Ans. 600790

(46) 8432
635
42160
25296
50592
Ans. 5354320

(47) 6874
829
61866
13748
54992
Ans. 5698546

(48) 2873
1823
8619
5746
22984
2873
Ans. 5237479

(49) 4786
3497
33502
43074
19144
14358
Ans. 16736642

(50) 87603
9865
438015
525618
700824
788427
Ans. 864203595

(51) 83457
6835
417285
250371
667656
500742
Ans. 570428595

(52) 31624
7138
252992
94872
31624
221368
Ans. 225732112

(53) 126
6
Ans. 756 dollars.

(54) 823
12
Ans. 9876 dollars.

(55) 675
13
2025
675
Ans. 8775 cents.

(56) 496
24
1984
992
Ans. 11904 cents.

(57) 152
28
1216
304
Ans. 4256 miles.

(58) 1760
209
15840
3520
Ans. 367840 yards.

(59) 365
24
1460
730
8760 hours.
8
Ans. 70080 miles.

(60) 1130
109
10170
1130
Ans. 123170 feet.

(61) 2029
1007
14203
2029
Ans. 2043203

(62) 80401
60007
562807
482406
Ans. 4824622807

(63) 101032
20001
101032
202064
Ans. 2020741032

ARTICLE 33.

(2)
124 or 124
6 8
744 992
4 3
Ans. 2976 2976

(3) 2395
8
19160
7
Ans. 134120

(4) 2873
9
25857
6
Ans. 155142

(5) 1673
9
15057
7
Ans. 105399

(6) 2874
9
25866
8
Ans. 206928

(7) 8074
12
96888
9
Ans. 871992

ARTICLE 34.

(1) 245
100
Ans. 24500

(2) 138
1000
Ans. 138000

(3) 428
10000
Ans. 4280000

(4) 872
100000
Ans. 87200000

ARTICLE 35.

(1) 2350
60
Ans. 141000

(2) 80300
450
4015
3212
Ans. 36135000

(3) 10240
3200
2048
3072
Ans. 32768000

(4) 9600
2400
384
192
Ans. 23040000

(5) 18001
26000
108006
36002
Ans. 468026000

(6) 8602
1030
25806
8602
Ans. 8860060

(7) 3007
9100
3007
27063
Ans. 27363700

(8) 80600
7002
1612
5642
Ans. 564361200

(9) 70302
80300
210906
562416
Ans. 5645250600

(10) 904000
10200
1808
904
Ans. 9220800000

DIVISION.

SUGGESTIONS TO TEACHERS.—Since division is only a short method of performing many subtractions, when the numbers to be subtracted are all equal to each other; and since we can only find the difference between numbers of the same kind, it is obvious that the divisor and dividend are always of the same name. Thus, if the dividend is dollars, the divisor must be dollars; if the dividend is an abstract number, the divisor must be the same.

Since the quotient shows *how many times* the same number has been subtracted, or, which is the same thing, how many times the divisor is contained in the dividend, it is always an abstract number. In the question, How many yards of ribbon at 2 cents a yard, can be bought for 6 cents? the answer is evidently 3 yards. It is not, however, to be inferred from this, that the quotient of 6 cents, divided by 2 cents, is 3 yards. The analysis of the question is, Since 2 cents buys 1 yard, and 6 cents is 3 times 2 cents, therefore 6 cents will buy three times as many yards as 2 cents, that is, 3 yards. (See Division, in Ray's Arithmetic, Second Book, page 43.)

Since we commence on the right in Addition, Subtraction, and Multiplication, it may be asked, why do we commence on the left in Division? The reason is, that remainders are often left in the higher orders, which must necessarily be carried to the lower orders.

ARTICLE 42.

(4) 3)60	**(5)** 2)400	**(6)** 3)6000	**(7)** 4)80000
Ans. 20	*Ans.* 200	*Ans.* 2000	*Ans.* 20000

ARTICLE 43.

(2) 3)693	**(3)** 4)848	**(4)** 2)4682
Ans. 231	*Ans.* 212	*Ans.* 2341
(5) 4)8408	**(6)** 3)36936	**(7)** 2)88468
Ans. 2102	*Ans.* 12312	*Ans.* 44234

ARTICLE 44.

(4) 3)462	**(5)** 5)1170	**(6)** 4)948
Ans. 154	*Ans.* 234	*Ans.* 237

ARTICLE 45.

(12) 2)8652	**(13)** 3)406235	**(14)** 4)675043
Ans. 4326	*Ans.* $135411\frac{2}{3}$	*Ans.* $168760\frac{3}{4}$
(15) 5)984275	**(16)** 6)258703	**(17)** 6)523408
Ans. 196855	*Ans.* $43117\frac{1}{6}$	*Ans.* $87234\frac{4}{6}$
(18) 7)8643275	**(19)** 8)6032520	**(20)** 9)9032706
Ans. $1234753\frac{4}{7}$	*Ans.* 754065	*Ans.* 1003634
(21) 10)1830024	**(22)** 11)603251	**(23)** 12)41674008
Ans. $183002\frac{4}{10}$	*Ans.* 54841	*Ans.* 3472834

(24) 3)894
Ans. 298 oranges.

(25) 4)140
Ans. 35 cents.

(26) 4)812
Ans. 203 barl.

(27) 9)423
Ans. $47

(28) 12)540
Ans. 45 years.

(29) 4)321276
Ans. 80319 gals.

(30) 8)1736
Ans. 217 barl.

(31) 7)734566
Ans. 104938 weeks.

(32) 11)495
Ans. 45

(33) 9)3582
Ans. 398

(34) 2)56
Ans. 28

(35) 2)3725
Ans. 1862$\frac{1}{2}$

(36) 3)147
Ans. 49

(37) 4)500
Ans. 125

(38) 5)1945
Ans. 389

(39) 6)4476
Ans. 746

(40) 7)2513
Ans. 359

(41) 8)5992
Ans. 749

(42) 9)8793
Ans. 977

(43) 10)1090
Ans. 109

(44) 11)4125
Ans. 375

(45) 12)5556
Ans. 463

(46) 4)144
3)36 each son's share.
Ans. 12

(47) 5)195
3)39
Ans. 13

ARTICLE 46.

(3)
13)4056(312 *Ans.*
39
15
13
26
26

(5)
14)11577(826$\frac{13}{14}$ *Ans.*
112
37
28
97
84
13

(6)
15)48690(3246 *Ans.*
45
36
30
69
60
90
90

(7)
23)1110960($48302\frac{14}{23}$ *Quotient.*
92
190
184
69
69
60
46
14

(8)
67)122878(1834 *Quotient.*
67
558
536
227
201
268
268

(9)
53)12412($234\frac{10}{53}$ *Quotient.*
106
181
159
222
212
10

(10)
72)146304(2032 *Quotient.*
144
230
216
144
144

(11)
54)47100($872\frac{12}{54}$ *Quotient.*
432
390
378
120
108
12

(12)
88)71104(808 *Quotient.*
704
704
704

(13)
66)43956(666 *Quotient.*
396
435
396
396
396

(14)
99)121900($1231\frac{31}{99}$ *Quotient.*
99
229
198
310
297
130
99
31

(15)

112)25312(226 *Quotient.*
224
291
224
672
672

(16)

123)381600(3102$\frac{54}{123}$ *Quotient.*
369
126
123
300
246
54

(17)

204)105672)518 *Quotient.*
1020
367
204
1632
1632

(18)

1234)600000(486$\frac{276}{1234}$ *Quotient.*
4936
10640
9872
7680
7404
276

(19)

4674)47263488(10112 *Quot.*
4674
5234
4674
5608
4674
9348
9348

(20)

2034)26497935(13027$\frac{1017}{2034}$ *Quot.*
2034
6157
6102
5593
4068
15255
14238
1017

(21)

9876)48905952(4952 *Quotient.*
39504
94019
88884
51355
49380
19752
19752

(22)

12345)4049160(328 *Quotient.*
37035
34566
24690
98760
98760

(23)
973)552160000(567482$\frac{14}{973}$ *Quotient.*
4865
6566
5838
7280
6811
4690
3892
7980
7784
1960
1946
14

(24)
15)3465(231 *Ans.*
30
46
45
15
15

(25)
26)364(14 days.
26
104
104

(26) 19)1083(57 dollars.
95
133
133

(27)
107)9523(89 bushels.
856
963
963

(28) 63)14868(236 hhd.
126
226
189
378
378

(29)
365)25000(68 dollars per day.
2190
3100
2920
$180 rem.

(30) 365)37960($104
365
1460
1460

(31)
1235)6571435(5321 *Ans.*
6175
3964
3705
2593
2470
1235
1235

(32)
405)1247400(3080 *Ans.*
1215
3240
3240
0

(33)
1006)10401000(10338$\frac{972}{1006}$ *Quotient.*
1006
3410
3018
3920
3018
9020
8048
972

ARTICLE 47.

(3) 9)2583
7)287
Ans. 41

(4) 4)6976
8)1744
Ans. 218

(5) 4)2744
7)686
Ans. 98

(6) 6)6145
7)1024—1 rem.
Ans. 146—2
6×2+1=13 rem.

(7) 11)19008
12)1728
Ans. 144

(8) 8)7840
8)980
Ans. 122—4
8×4=32 rem.

(9) 8)14771
9)1846—3
Ans. 205—1
8×1+3=11 rem.

(10) 9)10206
9)1134
Ans. 126

ARTICLE 48.

(2) 1|0)268|2
Ans. 268$\frac{2}{10}$

(3) 1|00)47|00
Ans. 47

(4) 1|00)372|01
Ans. 372$\frac{1}{100}$

(5) 1|00)462|50
Ans. 462$\frac{50}{100}$

(6) 1|00)620|34
Ans. 620$\frac{34}{100}$

(7) 1|000)18|003
Ans. 18$\frac{3}{1000}$

(8) 1|000)375|000
Ans. 375

ARTICLE 49.

(3) 4|000)73|005

Ans. $18\frac{1005}{4000}$

(4) 9|000)36|001

Ans. $4\frac{1}{9000}$

(5) 11|000)1078|000

Ans. 98

(6) 18|0)4016|7($223\frac{27}{180}$ *Quotient.*

```
36
--
41
36
--
 56
 54
 --
  2
```

(7)

21|00)9072|37($432\frac{37}{2100}$ *Quot.*

```
84
--
67
63
--
 42
 42
 --
```

(8)

64|00)3640|06($56\frac{5606}{6400}$ *Quot.*

```
320
---
440
384
---
 56
```

(9) 25|0000)7654|6037($306\frac{46037}{250000}$ *Quotient.*

```
 75
---
154
150
---
  4
```

(10) 634|00)435637|54($687\frac{7954}{63400}$ *Quotient.*

```
3804
----
5523
5072
----
 4517
 4438
 ----
   79
```

ARTICLE 50.

(1)
7895)434225(55 *Ans.*
39475
39475
39475

(2)
327)106929(327 *Ans.*
981
882
654
2289
2289

(3)
375)10741125(28643 *Ans.*
750
3241
3000
2411
2250
1612
1500
1125
1125

(4)
60352)63550656(1053 *Ans.*
60352
319865
301760
181056
181056

ARTICLE 56.

(1)
96
120
55
271

500
271
Ans. 229

(2)
243 1st.
61
304 2d.
79
225 3d.

243
304
225
772

1265
772
Ans. 493

(3)
157
264
305
97
123
946
614
Ans. 332

428
186
614

(4)
9503
586
4794
1234
850
16967

57068
16967
Ans. 40101

(5) $123078
8706
131784
5918
Ans. $125866

$237 spent in Jan.
301 spent in Feb.
538
5380
$5918 spent.

538
10 months left.
5380

(6) 72
4
288 w.
24
1152
576
Ans. 6912 lights.

(7)
yards.
73
9
657

yards.
88
12
1056
657
Ans. 1713

(8) days.
365
49
3285
1460
17885
99
160965
160965
Ans. 1770615

(9) 39 vols.
774
156
273
273
30186 pages.
2
60372 columns.
67
422604
362232
4044924 lines.
10
40449240 words.
4044924 lines.
47
28314468
16179696
190111428 letters.

(10)
86)31173(362$\frac{41}{86}$ *Ans.*
258
537
516
213
172
41

(11)
28
3
$
25)1400(56 *Ans.*
125
150
150

(12) 63 gallons.
5
15)315(21 *Ans.*
30
15
15

(13) 73900
70
214)73830(345 *Ans.*
642
963
856
1070
1070

(14) 148
56
204 sum.
92 diff.
408
1836
23)18768(816 *Ans.*
184
36
23
138
138

148
56
92

$
60
8
480
210
6)690
115 *Ans*

(15)
$
15
14
60
15
210

(16) 15)360(24=gallons filled in 1 hour.
30
60
60
2|0)36|0
18=gallons emptied in 1 hour.
24
18
6=gallons left in cistern per hour.

6)360
60 hours. *Ans.*

(17)
$30
70
2100
$6000
2100
25)3900(156 acres.
25
70 acres.
140
226 acres *Ans.*
125
150
150

(18) 222
125
Ans. 347

(19)
$$\begin{array}{r} 500 \\ 135 \\ \hline \textit{Ans. } 365 \end{array}$$

(20)
$$\begin{array}{r} 45 \\ 15 \\ \hline 225 \\ 45 \\ \hline 675 \\ 65 \\ \hline \textit{Ans. } 740 \end{array}$$

(21)
$$\begin{array}{r} 340 \\ 11 \\ \hline 9)351 \\ \hline 39 \\ 15 \\ \hline \textit{Ans. } 24 \end{array}$$

(22)
$$\begin{array}{r} 27 \\ 98 \\ \hline 125 \text{ diff.} \\ 246 \\ \hline \textit{Ans. } 371 \end{array}$$

ARTICLE 61.^{a}—CANCELLATION.

(2) $\frac{4\times\cancel{13}}{\cancel{13}}=4$ *Ans.*

(3) $\frac{\cancel{41}\times11}{\cancel{41}}=11$ *Ans.*

(6) $\frac{15\times\cancel{8}\,2}{\cancel{4}}=30$ *Ans.*

(7) $\frac{\overset{3}{\cancel{24}}\times4}{\cancel{8}}=12$ *Ans.*

(8) $\frac{37\times\cancel{15}\,3}{\cancel{5}}=111$ *Ans.*

(10) $\frac{36\times\cancel{5}}{\cancel{15}\,3}=12$ *Ans.*

(11) $\frac{14\times9}{6}=\frac{7\times\cancel{2}\times\cancel{3}\times3}{\cancel{2}\times\cancel{3}}=21$ *Ans.*

ARTICLE 61.^{b}

(1)

$$\frac{42\times25\times18}{21\times15}=\frac{\cancel{21}\times2\times\cancel{5}\times5\times\cancel{3}\times6}{\cancel{21}\times\cancel{5}\times\cancel{3}}=2\times5\times6=60 \textit{ Ans.}$$

(2) $\frac{2\,\cancel{10}\times23}{\cancel{5}}=2\times23=46$ *Ans.*

(3) $\frac{14\times15}{35}=\frac{2\times\cancel{7}\times3\times\cancel{5}}{\cancel{7}\times\cancel{5}}=6$ *Ans.*

(4)

$$\frac{21\times11\times6\times26}{13\times3\times14\times2}=\frac{\cancel{7}\times\cancel{3}\times11\times\cancel{2}\times3\times\cancel{13}\times\cancel{2}}{\cancel{13}\times\cancel{3}\times\cancel{7}\times\cancel{2}\times\cancel{2}}=11\times3=33$$

Ans.

$$(5)\ \frac{21\times15\times33\times8\times14\times17}{20\times34\times22\times27}$$

$$=\frac{7\times\cancel{3}\times\cancel{5}\times\cancel{3}\times\cancel{11}\times\cancel{3}\times\cancel{4}\times\cancel{2}\times7\times\cancel{2}\times\cancel{17}}{\cancel{4}\times\cancel{5}\times\cancel{17}\times\cancel{2}\times\cancel{11}\times\cancel{2}\times\cancel{3}\times\cancel{3}\times\cancel{3}}=7\times7=49$$

Ans.

(6)

$$\frac{21\times95\times6}{35\times9}=\frac{\cancel{7}\times\cancel{3}\times\cancel{5}\times19\times2\times\cancel{3}}{\cancel{7}\times\cancel{5}\times\cancel{3}\times\cancel{3}}=19\times2=38$$ *Ans.*

FEDERAL, OR U. S. MONEY.

ARTICLE 67.—NUMERATION.

(1) Eighteen dollars, sixty-two cents, five mills. (2) Twenty dollars, thirty-two cents, four mills. (3) Seventy-nine dollars, five cents. (4) Forty-six dollars, three mills. (5) Seventy dollars, one cent, five mills. (6) One hundred dollars, twenty-eight cents, three mills. (7) One hundred and fifty dollars and two mills. (8) One hundred dollars and three mills. (9) Six dollars and twelve cents. (10) Three dollars and six cents. (11) Four dollars and thirty-one cents. (12) Five dollars and forty-three cents. (13) Twenty-nine dollars. (14) One hundred dollars and three cents. (15) Twenty dollars and five cents. (16) Forty dollars and seven mills.

ARTICLE 68.—NOTATION.

The examples in the Arithmetic are all answered there. It is recommended to the teacher to exercise the pupils with additional examples, should it be found that they are not sufficiently acquainted with the subject, after going through with those in the book.

ARTICLE 71.—REDUCTION.

Since the operations in this section consist simply in adding ciphers or removing them, or erasing points or inserting them between the different denominations, it is deemed unnecessary to occupy space, as the whole solution, when presented to the eye, would consist in nothing more than writing down the question to be solved, and then placing the answer under it.

ARTICLE 72.—ADDITION.

(2) $ cts.	(3) $ cts. m.	(4) $ cts.
17.15 23.43 7.19 8.37 12.31	18.04 1 16.31 7 100.50 3 87.33 8	43.75 29.18 17.63 268.95 718.07
Ans. $68.45	*Ans.* $222.19 9	*Ans.* $1077.58

(5) $ cts. m.	(6) $ cts. m.	(7) $ cts. m.
200.00 0 43.87 5 56.93 7 8.50 0 2.31 3	504.06 3 420.19 7 105.50 0 304.00 5 888.45 5	5.07 0 30.20 3 100.00 5 60.02 0 700.01 1 1000.10 0 40.00 4 64.58 7
Ans. $311.62 5	*Ans.* $2222.22 0	*Ans.* $2000.00 0

ARTICLE 73.—SUBTRACTION.

(2) $ cts. m.	(3) $ cts.	(4) $ cts.	(5) $ cts.
29.34 2 17.26 5	46.28 17.75	20.05 5.50	3.00 .03
Ans. $12.07 7	*Ans.* $28.53	*Ans.* $14.55	*Ans.* $2.97

(6) $ cts. m.	(7) $ cts. m.	(8) $ cts. m.	(9) $ cts. m.
10.00 0 1	50.00 0 50 5	1000.00 0 1.01 1	1000.43 5 900.68 7
Ans. $9.99 9	*Ans.* $49.49 5	*Ans.* $998.98 9	*Ans.* $99.74 8

ARTICLE 74.—MULTIPLICATION.

(2) $ cts.	**(3)** $ cts. m.	**(4)** $ cts. m.	**(5)** $ cts.
7.83	12.09 3	23.01 8	35.14
8	9	16	53
Ans. $62.64	*Ans.* $108.83 7	138108	10542
		23018	17570
		Ans. $368.28 8	*Ans.* $1862.42

(6) $ cts.	**(7)** $ cts.	**(8)** cts. m.	**(9)** $ cts.
125.02	40.04	12 5	3.28
62	102	17	38
25004	8008	875	2624
75012	4004	125	984
Ans. $7751.24	*Ans.* $4084.08	*Ans.* $2.12 5	*Ans.* $124.64

(10) $ cts.	**(11)** $ cts.	**(12)**
1.06	5.75	465
338	38	7
848	4600	*Ans.* 32.55
318	1725	
318	*Ans.* $218.50	
Ans. $358.28		

Here, instead of multiplying 7 cents by 465, it amounts to the same thing to multiply 465 by 7, and is more convenient.

(13) cts.	**(14)** $ cts.	**(15)** $ cts.
34	5.67	2.69
89	24	169
306	2268	2421
272	1134	1614
Ans. $30.26	*Ans.* $136.08	269
		Ans. $454.61

(16)

$$\begin{array}{r} \$\ \text{cts.}\\ 1.25\\ 691\\ \hline 125\\ 1125\\ 750\\ \hline \textit{Ans.}\ \$863.75 \end{array}$$

(17)

$$\begin{array}{r} 73\\ 63\\ \hline 219\\ 438\\ \hline 4599\ \text{gal.}\\ 55\\ \hline 22995\\ 22995\\ \hline \textit{Ans.}\ \$2529.45 \end{array}$$

(18)

$$\begin{array}{r} 281\\ 4\\ \hline 1124\ \text{lb.}\\ 65\\ \hline 5620\\ 6744\\ \hline \textit{Ans.}\ \$73.060 \end{array}$$

(19)

$$\begin{array}{r} 35\\ 10\\ \hline 350\ \text{yd.}\\ 1\\ \hline \textit{Ans.}\ \$3.50 \end{array}$$

(20)

$$\begin{array}{r} 312\\ 11\\ \hline 3432\ \text{hours.}\\ 13\\ \hline 10296\\ 3432\\ \hline \textit{Ans.}\ \$446.16 \end{array}$$

(21)

$$\begin{array}{r} 18\\ 3\\ \hline \text{Bushels}\ 54 \end{array} \qquad \begin{array}{r} \$\ \text{cts.}\\ 1.25\\ 54\\ \hline 500\\ 625\\ \hline \textit{Ans.}\ \$67.50 \end{array}$$

(22)

$$\begin{array}{r} \$\ \text{cts. m.}\\ 10.00\ 1\\ 150\\ \hline 500050\\ 10001\\ \hline \textit{Ans.}\ 1500.15\ 0 \end{array}$$

(23)

$$\begin{array}{r} 17\\ 51\\ \hline 17\\ 85\\ \hline 867\ \text{lb.} \end{array} \qquad \begin{array}{r} \text{cts. m.}\\ 14\ \ 7\\ 867\\ \hline 1029\\ 882\\ 1176\\ \hline \textit{Ans.}\ \$127.44\ 9 \end{array}$$

ARTICLE 75.—DIVISION.

(4)

$$\begin{array}{r} 8)280\\ \hline \textit{Ans.}\ 35\ \text{yd.} \end{array}$$

(5)

$$\begin{array}{l} \text{cts. yd.}\\ 25)300(12\ \textit{Ans.}\\ 25\\ \hline 50\\ 50\\ \hline \end{array}$$

(6)

$$\begin{array}{l} \text{bbl.}\\ 805)16100(20\ \textit{Ans.}\\ 1610\\ \hline 0 \end{array}$$

(**7**) 75)1200(16 *Ans.*
75
450
450

(**8**) 1125)234000(208 bu. *Ans.*
2250
9000
9000

(**9**) 4)92.250
Ans. \$23.062+

(**10**) 8)57.500
Ans. \$7.187+

(**11**) \$ cts.
16)25.76(1.61 *Ans.*
16
97
96
16
16

(**12**) cts. m.
755)328.425(.43 5 *Ans.*
3020
2642
2265
3775
3775

(**13**) \$ cts. m.
313)800.000(2.55 5+ *Ans.*
626
1740
1565
1750
1565
1850
1565

(**14**) \$ cts. m. \$ cts. m.
133)10000.00 0(75.18 7+*Ans.*
931
690
665
250
133
1170
1064
1060
931
129

(**15**) \$ cts. m. \$ cts. m.
154)2705.01 0(17.56 5 *Ans.*
154
1165
1078
870
770
1001
924
770
770

(16)

$$\begin{array}{r}
25\\
15\\ \hline
125\\
25\\ \hline
\text{cts. cts.}\\
\text{lb. }375)6000(16\ \textit{Ans.}\\
375\\ \hline
2250\\
2250\\ \hline
\end{array}$$

(17)

$$\begin{array}{r}
235\\
8\\ \hline
\text{m.}\quad \text{m. cts.m.}\\
\text{lb. }1880)122200(65=6\ \ 5\ \textit{Ans.}\\
11280\\ \hline
9400\\
9400\\ \hline
\end{array}$$

ARTICLE 76.

(1)

$$\begin{array}{r}
\text{\$ cts.}\\
47.50\\
38.45\\
15.47\\
19.43\\ \hline
\textit{Ans.}\ \$120.85
\end{array}$$

(2)

$$\begin{array}{r}
\text{\$ cts.}\\
35.25\\
23.75\\ \hline
59.00\\
59.00\\
1.00\\ \hline
\textit{Ans.}\ \$119.00
\end{array}$$

(3)

$$\begin{array}{r}
\text{\$ cts.}\\
18.38\\
81.62\\ \hline
100.00\\
100.00\\
100.00\\ \hline
\textit{Ans}\ \$300.00
\end{array}$$

(4)

$$\begin{array}{r}
\text{\$ cts.}\\
.75\\
.35\\
.50\\
1.50\\ \hline
\$3.10
\end{array}
\qquad
\begin{array}{r}
\text{\$ cts.}\\
5.00\\
3.10\\ \hline
\textit{Ans.}\ \$1.90
\end{array}$$

(5)

$$\begin{array}{r}
\text{\$ cts.}\\
8.10\\
1.65\\
6.00\\
4.00\\ \hline
\$19.75
\end{array}
\qquad
\begin{array}{r}
\text{cts.}\\
.75\\
8\\ \hline
\$6.00
\end{array}
\qquad
\begin{array}{r}
\text{\$ cts.}\\
20.00\\
19.75\\ \hline
\textit{Ans.}\ .25\text{ cts.}
\end{array}$$

(6)

$$\begin{array}{r}
\text{\$ cts.}\\
50.00\\
30.50\\ \hline
19.50\\
6\\ \hline
\textit{Ans.}\ \$117.00
\end{array}$$

(7)

$$\begin{array}{r}
\text{\$ cts.}\\
3.85\\
1.25\\
2.50\\
1.50\\ \hline
9.10\\ \hline
\text{\$ cts.}\\
21.75\\
9.10\\ \hline
\textit{Ans.}\ \$12.65
\end{array}$$

(8)

$$\begin{array}{r}
\text{\$ cts.}\\
37.06\\
200.85\\
400.00\\
236.75\\
124.34\\ \hline
999.00\\
889.25\\ \hline
\textit{Ans.}\ \$109.75
\end{array}$$

(9)

$$\begin{array}{r}
143\\
.13\\ \hline
429\\
143\\ \hline
\$18.59\\
12.60\\ \hline
\textit{Ans.}\ \$5.99
\end{array}$$

(10)

$$\begin{array}{r}
435\\
45\\ \hline
2175\\
1740\\ \hline
\$195.75\\ \hline
\$400.00\\
195.75\\ \hline
\textit{Ans.}\ \$204.25
\end{array}$$

(11)
365
.65
1825
2190
$237.25
$400.00
237.25
Ans. $162.75

(12)
21
3
63
35
315
189
Ans. $22.05

(13)
19
4
76
23
228
152
Ans. $17.48

(14) $ cts.
2000.00
163.75
5)1836.25
5)367.25
Ans. $73.45

(15) $ cts.
4)516.00
4)129.00
cts.
43)32.25(75 *Ans.*
301
215
215

(16) $ cts.
4|0)9|0.00
10)2.25
Ans. cts. 22 5 m.

(17) $ cts.
22=2×11 1000.00
500.00
2)1500.00
11)750.00
Ans. $68.18+

(18)

	$ cts.
9 lbs. at 16 cts.	$1.44
4 lbs. at $1.25	5.00
45 lbs. at 9 cts.	4.05
17 lbs. at 13 cts.	2.21
Ans.	$12.70

(19)

	$ cts.
8 yards at $1.10	8.80
18 yards at 25 cts.	4.50
25 yards at 15 cts.	3.75
12 yards at 35 cts.	4.20
6 yards at 65 cts.	3.90
Ans.	$25.15

(20)

	$ cts.
5 copies at 35 cts.	1.75
12 dozen at $10.50	126.00
6 M. at $1.60	9.60
5 quires at 25 cts.	1.25
3 copies at $4.50	13.50
Ans.	$152.10

(21)

	$ cts.
27 copies at 19 cts.	5.13
25 " " 27 "	6.75
8 " " $4.50	36.00
27 " " $2.25	60.75
18 " " $1.50	27.00
75 " " 31 cts.	23.25
Ans.	$158.88

COMPOUND NUMBERS.

In the remaining part of this work, instead of writing out the solution in full, we have merely indicated the operations to be performed. When a person understands the elementary rules, and the meaning of the signs, the *method* of solution is just as easily understood, as if written out at length. Pupils, however, should be taught to work the questions, after the manner given in the book.

In Reduction, pupils should be frequently required to give the reason for multiplying or dividing by the numbers used: also to point out the name of the remainder, and give the reason for it.

ARTICLE 81.—DRY MEASURE.

(11) $\overset{\text{bu.}}{2}\times4\times8\times2=\overset{\text{pt.}}{128}$ *Ans.*

(12) $\overset{\text{pk.}}{12}\times8\times2=\overset{\text{pt.}}{192}$ *Ans.*

(13) $\overset{\text{bu.}}{8}\times4\times8=\overset{\text{qt.}}{256}$ *Ans.*

(14) $\overset{\text{bu.}}{1}\times4+\overset{\text{pk.}}{1}=\overset{\text{pk.}}{5}$: $\overset{\text{pk.}}{5}\times8\times2=\overset{\text{pt.}}{80}$ *Ans.*

(15) $\overset{\text{bu.}}{2}\times4=\overset{\text{pk.}}{8}$: $\overset{\text{pk.}}{8}\times8+\overset{\text{qt.}}{2}=\overset{\text{qt.}}{66}$: $\overset{\text{qt.}}{66}\times2=\overset{\text{pt.}}{132}$ *Ans.*

(16) $\overset{\text{bu.}}{4}\times4+\overset{\text{pk.}}{2}=\overset{\text{pk.}}{18}$: $\overset{\text{pk.}}{18}\times8+\overset{\text{qt.}}{1}=\overset{\text{qt.}}{145}$: $\overset{\text{qt.}}{145}\times2=\overset{\text{pt.}}{290}$ *Ans.*

(17) $\overset{\text{bu.}}{7}\times4+\overset{\text{pk.}}{3}=\overset{\text{pk.}}{31}$: $\overset{\text{pk.}}{31}\times8+\overset{\text{qt.}}{7}=\overset{\text{qt.}}{255}$: $\overset{\text{qt.}}{255}\times2+\overset{\text{pt.}}{1}=\overset{\text{pt.}}{511}$ *Ans.*

(18) $\overset{\text{bu.}}{3}\times4=\overset{\text{pk.}}{12}$: $\overset{\text{pk.}}{12}\times8=\overset{\text{qt.}}{96}$: $\overset{\text{qt.}}{96}\times2+\overset{\text{pt.}}{1}=\overset{\text{pt.}}{193}$ *Ans.*

(19) $\overset{\text{pt.}}{384}\div2=\overset{\text{qt.}}{192}$: $\overset{\text{qt.}}{192}\div8=\overset{\text{pk.}}{24}$: $\overset{\text{pk.}}{24}\div4=\overset{\text{bu.}}{6}$ *Ans.*

(20) $\overset{\text{pt.}}{47}\div2=\overset{\text{qt.}}{23}\ \overset{\text{pt.}}{1}$: $\overset{\text{qt.}}{23}\div8=\overset{\text{pk.}}{2}\ \overset{\text{qt.}}{7}$. *Ans.* $\overset{\text{pk.}}{2}\ \overset{\text{qt.}}{7}\ \overset{\text{pt.}}{1}$

(21) $\overset{\text{pt.}}{95}\div2=\overset{\text{qt.}}{47}\ \overset{\text{pt.}}{1}$: $\overset{\text{qt.}}{47}\div8=\overset{\text{pk.}}{5}\ \overset{\text{qt.}}{7}$: $\overset{\text{pk.}}{5}\div4=\overset{\text{bu.}}{1}\ \overset{\text{pk.}}{1}$: collecting the different remainders, the *Ans.* is, 1 bu. 1 pk. 7 qt. 1 pt.

pt. qt. qt. pk. qt. pk. bu. pk. bu.pk.qt.
(22) 508÷2=254 : 254÷8=31 6 : 31÷4=7 3. *Ans.* 7 3 6

ARTICLE 82.—TROY WEIGHT.

lb. gr.
(2) 4×12×20×24=23040 *Ans.*

lb. oz. oz.
(3) 5×12+4=64 *Ans.*

lb. oz. oz. oz. pwt. pwt.
(4) 9×12+3=111 : 111×20+5=2225 *Ans.*

lb. oz. oz. oz. pwt. pwt.
(5) 14×12+11=179 : 179×20+19=3599 :
pwt. gr. gr.
3599×24+23=86399 *Ans.*

lb. oz. oz. oz. pwt. pwt.
(6) 8×12+9=105 : 105×20+13=2113 :
pwt. gr. gr.
2113×24+17=50729 *Ans.*

gr. pwt. gr.
(7) 171÷24=7 3 *Ans*

gr. pwt. gr. pwt. oz. pwt. oz. pwt. gr.
(8) 505÷24=21 1 : 21÷20=1 1. *Ans.* 1 1 1.

gr. pwt. gr. pwt. oz. pwt. oz. lb. oz.
(9) 12530÷24=522 2 : 522÷20=26 2 : 26÷12=2 2.
lb. oz. pwt. gr.
Ans. 2 2 2 2.

pwt. oz. pwt. oz. lb. oz. lb. oz. pwt.
(10 805÷20=40 5 : 40÷12=3 4. *Ans.* 3 4 5.

gr. pwt. gr. pwt. oz. pwt. oz. lb.oz.
(11) 25591÷24=1066 7 : 1066÷20=53 6 : 53÷12=4 5.
lb. oz. pwt. gr.
Ans. 4 5 6 7.

ARTICLE 83.—APOTHECARIES' WEIGHT.

℔ gr
(1) 3×12×8×3×20=17280 *Ans.*

(2) $\overset{\text{℔}}{4}\times12=\overset{\text{℥}}{48}$: $\overset{\text{℥}}{48}\times8+\overset{\text{ʒ}}{5}=\overset{\text{ʒ}}{389}$: $\overset{\text{ʒ}}{389}\times3=\overset{\text{℈}}{1167}$:
$\overset{\text{℈}}{1167}\times20+\overset{\text{gr.}}{2}=\overset{\text{gr.}}{23342}$ *Ans.*

(3) $\overset{\text{℔}}{7}\times12=\overset{\text{℥}}{84}$: $\overset{\text{℥}}{84}\times8=\overset{\text{ʒ}}{672}$: $\overset{\text{ʒ}}{672}\times3+\overset{\text{℈}}{2}=\overset{\text{℈}}{2018}$ *Ans.*

(4) $\overset{\text{℔}}{7}\times12+\overset{\text{℥}}{2}=\overset{\text{℥}}{86}$: $\overset{\text{℥}}{86}\times8=\overset{\text{ʒ}}{688}$: $\overset{\text{ʒ}}{688}\times3+\overset{\text{℈}}{1}=\overset{\text{℈}}{2065}$:
$\overset{\text{℈}}{2065}\times20=\overset{\text{gr.}}{41300}$ *Ans.*

(5) $\overset{\text{℥}}{67}\div12=\overset{\text{℔}}{5}\ \overset{\text{℥}}{7}$ *Ans.*

(6) $\overset{\text{ʒ}}{431}\div8=\overset{\text{℥}}{53}\ \overset{\text{ʒ}}{7}$: $\overset{\text{℥}}{53}\div12=\overset{\text{℔}}{4}\ \overset{\text{℥}}{5}$. *Ans.* $\overset{\text{℔}}{4}\ \overset{\text{℥}}{5}\ \overset{\text{ʒ}}{7}$.

(7) $\overset{\text{℈}}{975}\div3=\overset{\text{ʒ}}{325}$: $\overset{\text{ʒ}}{325}\div8=\overset{\text{℥}}{40}\ \overset{\text{ʒ}}{5}$: $\overset{\text{℥}}{40}\div12=\overset{\text{℔}}{3}\ \overset{\text{℥}}{4}$. *Ans.* $\overset{\text{℔}}{3}\ \overset{\text{℥}}{4}\ \overset{\text{ʒ}}{5}$.

(8) $\overset{\text{gr.}}{6321}\div20=\overset{\text{℈}}{316}\ \overset{\text{gr.}}{1}$: $\overset{\text{℈}}{316}\div3=\overset{\text{ʒ}}{105}\ \overset{\text{℈}}{1}$: $\overset{\text{ʒ}}{105}\div8=\overset{\text{℥}}{13}\ \overset{\text{ʒ}}{1}$:
$\overset{\text{℥}}{13}\div12=\overset{\text{℔}}{1}\ \overset{\text{℥}}{1}$. *Ans.* $\overset{\text{℔}}{1}\ \overset{\text{℥}}{1}\ \overset{\text{ʒ}}{1}\ \overset{\text{℈}}{1}\ \overset{\text{gr.}}{1}$.

(9) $\overset{\text{gr.}}{30941}\div20=\overset{\text{℈}}{1547}\ \overset{\text{gr.}}{1}$: $\overset{\text{℈}}{1547}\div3=\overset{\text{ʒ}}{515}\ \overset{\text{℈}}{2}$: $\overset{\text{ʒ}}{515}\div8=\overset{\text{℥}}{64}\ \overset{\text{ʒ}}{3}$:
$\overset{\text{℥}}{64}\div12=\overset{\text{℔}}{5}\ \overset{\text{℥}}{4}$. *Ans.* $\overset{\text{℔}}{5}\ \overset{\text{℥}}{4}\ \overset{\text{ʒ}}{3}\ \overset{\text{℈}}{2}\ \overset{\text{gr.}}{1}$.

(10) $\overset{\text{gr.}}{29239}\div20=\overset{\text{℈}}{1461}\ \overset{\text{gr.}}{19}$: $\overset{\text{℈}}{1461}\div3=\overset{\text{ʒ}}{487}$: $\overset{\text{ʒ}}{487}\div8=\overset{\text{℥}}{60}\ \overset{\text{ʒ}}{7}$:
$\overset{\text{℥}}{60}\div12=\overset{\text{℔}}{5}$. *Ans.* $\overset{\text{℔}}{5}\ \overset{\text{ʒ}}{7}\ \overset{\text{gr.}}{19}$.

ARTICLE 84.—AVOIRDUPOIS WEIGHT.

(1) $\overset{\text{cwt.}}{2}\times4\times25=\overset{\text{lb.}}{200}$ *Ans.* **(2)** $\overset{\text{cwt.}}{3}\times4+\overset{\text{qr.}}{3}=\overset{\text{qr.}}{15}$: $\overset{\text{qr.}}{15}\times25=\overset{\text{lb.}}{375}$ [*Ans.*

T. cwt. cwt. cwt. lb.
(3) 1×20+2=22 : 22×4×25=2200 *Ans.*

T. cwt. cwt. qr. qr. qr. lb.
(4) 3×20=60 : 60×4+3=243 : 243×25=6075 *Ans.*

cwt. qr. qr. qr. lb. lb.
(5) 4×4+1=17 : 17×25+19=444 *Ans.*

T. cwt. cwt. qr. qr. qr. lb. lb.
(6) 5×20=100 : 100×4+3=403 : 403×25+15=10090
[*Ans.*

cwt. qr. qr. qr. lb. lb. lb. oz. oz.
(7) 2×4+3=11 : 11×25+2=277 : 277×16+12=4444
[*Ans.*

cwt. qr. qr. lb. lb lb. oz.
(8) 2×4=8 : 8×25+17=217 : 217×16=3472 :
oz. dr. dr.
3472×16+3=55555 *Ans.*

T. cwt. cwt. cwt. qr. qr. lb. lb.
(9) 1×20+6=26 : 26×4=104 : 104×25+4=2604 :
lb. oz. oz. oz. dr. dr.
2604×16+2=41666 : 41666×16+10=666666 *Ans.*

lb. qr. lb. qr. cwt. cwt. lb.
(10) 4803÷25=192 3 : 192÷4=48. *Ans.* 48 3.

lb. qr. qr. cwt. cwt. T. cwt.
(11) 22400÷25=896 : 896÷4=224 : 224÷20=11 4 *Ans.*

dr. oz. oz. lb.
(12) 2048000÷16=128000 : 128000÷16=8000 :
lb. qr. qr. cwt. cwt. T.
8000÷25=320 : 320÷4=80 : 80÷20=4 *Ans.*

dr. oz. dr. oz. lb. oz.
(13) 64546÷16=4034 2 : 4034÷16=252 2 :
lb. qr. lb. qr. cwt. qr. cwt. qr. lb. oz. dr.
252÷25=10 2 : 10÷4=2 2. *Ans.* 2 2 2 2 2.

oz. lb. oz. lb. qr. qr. cwt. qr.
(14) 97203÷16=6075 3 : 6075÷25=243 : 243÷4=60 3 :
cwt. T. T. qr. oz.
60÷20=3. *Ans.* 3 3 3.

dr. oz. oz. lb. oz.
(15) 544272÷16=34017 : 34017÷16=2126 1 :

lb. qr. lb. qr. cwt. qr. cwt. T. cwt.
2126÷25=85 1 : 85÷4=21 1 : 21÷20=1 1.

T. cwt. qr. lb. oz.
Ans. 1 1 1 1 1.

lb. lb. lb. qr. lb. qr. cwt.qr.
(16) 18×52=936 : 936÷25=37 11 : 37÷4=9 1.

cwt. qr. lb.
Ans. 9 1 11.

ARTICLE 85.—LONG MEASURE.

yd. ft. ft. ft. in. in.
(1) 2×3+2=8 : 8×12+7=103 *Ans.*

yd. ft. ft. in. in.
(2) 7×3=21 : 21×12+11=263 *Ans*

mi. fur. fur. rd.
(3) 12×8=96 : 96×40=3840 *Ans.*

mi. fur. fur. fur. rd.
(4) 7×8+6=62 : 62×40=2480 *Ans.*

mi. fur. fur. rd. rd.
(5) 9×8=72 : 72×40+31=2911 *Ans.*

in. ft. in. ft. yd. ft. yd. ft. in.
(6) 133÷12=11 1 : 11÷3=3 2 *Ans.* 3 2 1

in. ft. in. ft. yd. yd. in.
(7) 181÷12=15 1 : 15÷3=5. *Ans.* 5 1.

rd. fur. fur. mi.
(8) 2240÷40=56 : 56÷8=7 *Ans.*

rd. fur. fur. mi. fur.
(9) 2200÷40=55 : 55÷8=6 7 *Ans.*

ARTICLE 88.—LAND MEASURE.

(1) $\overset{\text{sq. yd.}}{8}\times 9\times 144=\overset{\text{sq. in.}}{10368}$ *Ans.* (2) $\overset{\text{A.}}{4}\times 4\times 40=\overset{\text{P.}}{640}$ *Ans.*

(3) $\overset{\text{sq. mi.}}{1}\times 640\times 4\times 40=\overset{\text{P.}}{102400}$ *Ans.*

(4) $\overset{\text{sq.yd.}}{2}\times 9+\overset{\text{sq.ft.}}{3}=\overset{\text{sq.ft.}}{21}$: $\overset{\text{sq.ft.}}{21}\times 144=\overset{\text{sq. in.}}{3024}$ *Ans.*

(5) $\overset{\text{A}}{5}\times 4+\overset{\text{R.}}{2}=\overset{\text{R.}}{22}$: $\overset{\text{R.}}{22}\times 40+\overset{\text{P.}}{20}=\overset{\text{P.}}{900}$ *Ans.*

(6) $\overset{\text{P.}}{960}\div 40=\overset{\text{R.}}{24}$: $\overset{\text{R.}}{24}\div 4=\overset{\text{A.}}{6}$ *Ans.*

(7) $\overset{\text{sq. in.}}{3888}\div 144=\overset{\text{sq. ft.}}{27}$: $\overset{\text{sq. ft.}}{27}\div 9=\overset{\text{sq. yd.}}{3}$ *Ans.*

(8) $\overset{\text{P.}}{243}\div 40=\overset{\text{R.}}{6}\ \ \overset{\text{P.}}{3}$: $\overset{\text{R.}}{6}\div 4=\overset{\text{A.}}{1}\ \ \overset{\text{R.}}{2}.$ *Ans.* $\overset{\text{A.}}{1}\ \ \overset{\text{R.}}{2}\ \ \overset{\text{P.}}{3}.$

(9) $\overset{\text{P.}}{603}\div 40=\overset{\text{R.}}{15}\ \ \overset{\text{P.}}{3}$: $\overset{\text{R.}}{15}\div 4=\overset{\text{A.}}{3}\ \ \overset{\text{R.}}{3}.$ *Ans.* $\overset{\text{A.}}{3}\ \ \overset{\text{R.}}{3}\ \ \overset{\text{P.}}{3}.$

(10) $\overset{\text{sq. in.}}{4176}\div 144=\overset{\text{sq. ft.}}{29}$: $\overset{\text{sq. ft.}}{29}\div 9=\overset{\text{sq. yd.}}{3}\ \ \ \overset{\text{sq. ft.}}{2}$ *Ans.*

ARTICLE 89.

(2) $\overset{\text{ft.}}{16}\times\overset{\text{ft.}}{12}=\overset{\text{sq. ft.}}{192}$ *Ans.* (3) $\overset{\text{yd.}}{5}\times\overset{\text{yd.}}{4}=\overset{\text{sq. yd.}}{20}$ *Ans.*

(4) $\overset{\text{ft.}}{18}\div 3=\overset{\text{yd.}}{6}$: $\overset{\text{ft.}}{12}\div 3=\overset{\text{yd.}}{4}$: $\overset{\text{ft.}}{21}\div 3=\overset{\text{yd.}}{7}$: $\overset{\text{ft.}}{15}\div 3=\overset{\text{yd.}}{5}$:
$6\times 4=\overset{\text{sq. yd.}}{24}$: $7\times 5=\overset{\text{sq. yd.}}{35}$: $24+35=\overset{\text{sq. yd.}}{59}$ *Ans.*

(5) $\overset{\text{ft.}}{18}\times\overset{\text{ft.}}{14}=\overset{\text{sq. ft.}}{252}$: $\overset{\text{sq. ft.}}{252}\div 9=\overset{\text{sq. yd.}}{28}$ *Ans.*

(6) $\overset{\text{rd.}}{35}\times\overset{\text{rd.}}{32}=\overset{\text{P.}}{1120}$: $\overset{\text{P.}}{1120}\div 40=\overset{\text{R.}}{28}$: $\overset{\text{R.}}{28}\div 4=\overset{\text{A.}}{7}$ *Ans.*

ARTICLE 90.

(1) $\overset{\text{sq. ft.}}{132} \div \overset{\text{ft.}}{11} = \overset{\text{ft.}}{12}$ *Ans.* **(2)** $\overset{\text{sq.yd.}}{30} \times 9 = \overset{\text{sq. ft.}}{270} : \overset{\text{sq. ft.}}{270} \div \overset{\text{ft.}}{18} = \overset{\text{ft.}}{15}$ *Ans.*

(3) $\overset{\text{A.}}{9} \times 4 \times 40 = \overset{\text{P.}}{1440} : \overset{\text{P.}}{1440} \div 45 = \overset{\text{rd.}}{32}$ *Ans.*

(4) $\overset{\text{A.}}{21} \times 4 \times 40 = \overset{\text{P.}}{3360} : \overset{\text{P.}}{3360} \div 35 = \overset{\text{rd.}}{96}$ *Ans.*

ARTICLE 93.—CUBIC MEASURE.

(1) $\overset{\text{cu. yd.}}{2} \times 27 \times 1728 = \overset{\text{cu. in.}}{93312}$ *Ans.* **(2)** $\overset{\text{C.}}{28} \times 128 = \overset{\text{cu. ft.}}{3584}$ *Ans.*

(3) $\overset{\text{C.}}{34} \times 8 = \overset{\text{C. ft.}}{272}$ *Ans.* **(4)** $\overset{\text{C.}}{1} \times 128 \times 1728 = \overset{\text{cu. in.}}{221184}$ *Ans.*

(5) $\overset{\text{cu. in.}}{63936} \div 1728 = \overset{\text{cu. ft.}}{37} : \overset{\text{cu.ft.}}{37} \div 27 = \overset{\text{cu.yd.}}{1}\ \ \overset{\text{cu.ft.}}{10}$ *Ans.*

(6) $\overset{\text{cu. in.}}{492480} \div 1728 = \overset{\text{cu. ft.}}{285} : \overset{\text{cu. ft.}}{285} \div 40 = \overset{\text{tn.}}{7}\ \ \overset{\text{cu.ft.}}{5}$ *Ans.*

(7) $\overset{\text{ft.}}{8} \times \overset{\text{ft.}}{5} \times \overset{\text{ft.}}{4} = \overset{\text{cu.ft.}}{160}$ *Ans.* **(8)** $\overset{\text{yd.}}{8} \times \overset{\text{yd.}}{5} \times \overset{\text{yd.}}{2} = \overset{\text{cu.yd.}}{80}$ *Ans.*

(9) $\overset{\text{ft.}}{18} \times \overset{\text{ft.}}{15} \times \overset{\text{ft.}}{7} = \overset{\text{cu.ft.}}{1890} : \overset{\text{cu.ft.}}{1890} \div 27 = \overset{\text{cu.yd.}}{70}$ *Ans.*

(10) $\overset{\text{ft.}}{40} \times \overset{\text{ft.}}{12} \times \overset{\text{ft.}}{8} = \overset{\text{cu.ft.}}{3840} : \overset{\text{cu.ft.}}{3840} \div 128 = \overset{\text{C.}}{30}$ *Ans.*

ARTICLE 94.—CLOTH MEASURE.

(1) $\overset{\text{yd.}}{19} \times 4 \times 4 = \overset{\text{na.}}{304}$ *Ans.* **(2)** $\overset{\text{E.Fl.}}{14} \times 3 \times 4 = \overset{\text{na.}}{168}$ *Ans.*

(3) $\overset{\text{yd.}}{5} \times 4 + \overset{\text{qr.}}{2} = \overset{\text{qr.}}{22} : \overset{\text{qr.}}{22} \times 4 + \overset{\text{na.}}{3} = \overset{\text{na.}}{91}$ *Ans.*

(4) $\overset{\text{E. En.}}{13} \times 5 + \overset{\text{qr.}}{1} = \overset{\text{qr.}}{66} : \overset{\text{qr.}}{66} \times 4 = \overset{\text{na.}}{264}$ *Ans.*

E. Fr. qr. qr. qr. na. na.
(5) 23×6+3=141 : 141×4+2=566 *Ans.*

na. qr. na. qr. yd. qr. yd. qr. na.
(6) 159÷4=39 3 : 39÷4=9 3. *Ans.* 9 3 3.

na. qr. na. qr. E.Fr. na. E.Fr. qr. na.
(7) 287÷4=71 3 : 71÷6=11 5. *Ans.* 11 5 3.

yd. qr. qr. E.Fl.
(8) 6×4=24 : 24÷3=8 *Ans.*

yd. qr. qr. qr. E.Fl.
(9) 9×4+3=39 : 39÷3=13 *Ans.*

yd. qr. qr. qr. E.Fl. qr.
(10) 12×4+1=49 : 49÷3=16 1 *Ans.*

E.Fl. qr. qr. yd. qr.
(11) 37×3=111 : 111÷4=27 3 *Ans.*

E.En. qr. qr. E.Fr.
(12) 36×5=180 : 180÷6=30 *Ans.*

E.En. qr. qr. qr. E.Fr.
(13) 22×5+4=114 : 114÷6=19 *Ans.*

E.En. qr. qr. yd. qr.
(14) 47×5=235 : 235÷4=58 3 *Ans.*

Article 95.—Liquid Measure.

gal. pt. gal. gi.
(1) 17×4×2=136 *Ans.* **(2)** 13×4×2×4=416 *Ans.*

hhd. pt.
(3) 2×63×4×2=1008 *Ans.*

T. gi.
(4) 5×4×63×4×2×4=40320 *Ans.*

T. hhd. hhd. hhd. gal.
(5) 3×4+3=15 : 15×63=945 *Ans.*

(6) $\overset{\text{hhd.}}{1}\times 63+\overset{\text{gal.}}{60}=\overset{\text{gal.}}{123} : \overset{\text{gal.}}{123}\times 4=\overset{\text{qt.}}{492} : \overset{\text{qt.}}{492}\times 2+\overset{\text{pt.}}{1}=\overset{\text{pt.}}{985}$ *Ans.*

(7) $\overset{\text{hhd.}}{2}\times 63+\overset{\text{gal.}}{17}=\overset{\text{gal.}}{143} : \overset{\text{gal.}}{143}\times 4+\overset{\text{qt.}}{3}=\overset{\text{qt.}}{575} : \overset{\text{qt.}}{575}\times 2\times 4=\overset{\text{gi.}}{4600}$
[*Ans.*

(8) $\overset{\text{T.}}{2}\times 4=\overset{\text{hhd.}}{8} : \overset{\text{hhd.}}{8}\times 63+\overset{\text{gal.}}{62}=\overset{\text{gal.}}{566} : \overset{\text{gal.}}{566}\times 4\times 2+\overset{\text{pt.}}{1}=\overset{\text{pt.}}{4529}$:
$\overset{\text{pt.}}{4529}\times 4=\overset{\text{gi.}}{18116}$ *Ans.*

(9) $\overset{\text{gi.}}{96}\div 4=\overset{\text{pt.}}{24} : \overset{\text{pt.}}{24}\div 2=\overset{\text{qt.}}{12} : \overset{\text{qt.}}{12}\div 4=\overset{\text{gal.}}{3}$ *Ans.*

(10) $\overset{\text{gi.}}{6048}\div 4=\overset{\text{pt.}}{1512} : \overset{\text{pt.}}{1512}\div 2=\overset{\text{qt.}}{756} : \overset{\text{qt.}}{756}\div 4=\overset{\text{gal.}}{189}$:
$\overset{\text{gal.}}{189}\div 63=\overset{\text{hhd.}}{3}$ *Ans.*

(11) $\overset{\text{gi.}}{32256}\div 4=\overset{\text{pt.}}{8064} : \overset{\text{pt.}}{8064}\div 2=\overset{\text{qt.}}{4032} : \overset{\text{qt.}}{4032}\div 4=\overset{\text{gal.}}{1008}$:
$\overset{\text{gal.}}{1008}\div 63=\overset{\text{hhd.}}{16} : \overset{\text{hhd.}}{16}\div 4=\overset{\text{T.}}{4}$ *Ans.*

(12) $\overset{\text{gi.}}{4050}\div 4=\overset{\text{pt.}}{1012}\ \overset{\text{gi.}}{2} : \overset{\text{pt.}}{1012}\div 2=\overset{\text{qt.}}{506} : \overset{\text{qt.}}{506}\div 4=\overset{\text{gal.}}{126}\ \overset{\text{qt.}}{2}$:
$\overset{\text{gal.}}{126}\div 63=\overset{\text{hhd.}}{2}.$ *Ans.* $\overset{\text{hhd.}}{2}\ \overset{\text{qt.}}{2}\ \overset{\text{gi.}}{2}.$

(13) $\overset{\text{gi.}}{30339}\div 4=\overset{\text{pt.}}{7584}\ \overset{\text{gi.}}{3} : \overset{\text{pt.}}{7584}\div 2=\overset{\text{qt.}}{3792} : \overset{\text{qt.}}{3792}\div 4=\overset{\text{gal.}}{948}$:
$\overset{\text{gal.}}{948}\div 63=\overset{\text{hhd.}}{15}\ \overset{\text{gal.}}{3} : \overset{\text{hhd.}}{15}\div 4=\overset{\text{T.}}{3}\ \overset{\text{hhd.}}{3}.$ *Ans.* $\overset{\text{T.}}{3}\ \overset{\text{hhd.}}{3}\ \overset{\text{gal.}}{3}\ \overset{\text{gi.}}{3}.$

(14) $\overset{\text{gi.}}{10125}\div 4=\overset{\text{pt.}}{2531}\ \overset{\text{gi.}}{1} : \overset{\text{pt.}}{2531}\div 2=\overset{\text{qt.}}{1265}\ \overset{\text{pt.}}{1} : \overset{\text{qt.}}{1265}\div 4=\overset{\text{gal.}}{316}\ \overset{\text{qt.}}{1}$:
$\overset{\text{gal.}}{316}\div 63=\overset{\text{hhd.}}{5}\ \overset{\text{gal.}}{1} : \overset{\text{hhd.}}{5}\div 4=\overset{\text{T.}}{1}\ \overset{\text{hhd.}}{1}.$ *Ans.* $\overset{\text{T.}}{1}\ \overset{\text{hhd.}}{1}\ \overset{\text{gal.}}{1}\ \overset{\text{qt.}}{1}\ \overset{\text{pt.}}{1}\ \overset{\text{gi.}}{1}.$

pn. gi.
(15) 3×84×4×2×4=8064 *Ans.*

p. qt.
(16) 5×126×4=2520 *Ans.*

pt. qt. qt. gal. gal. tr.
(17) 5712÷2=2856 : 2856÷4=714 : 714÷42=17 *Ans.*

ARTICLE 96.—BEER MEASURE.

hhd. pt.
(1) 4×54×4×2=1728 *Ans.*

hhd. gal. gal. qt. qt. qt. pt.
(2) 7×54=378 : 378×4+3=1515 : 1515×2=3030 *Ans.*

pt. qt. qt. gal. gal. bl. gal.
(3) 1000÷2=500 : 500÷4=125 : 125÷36=3 17. *Ans.*

pt. qt. pt. qt. gal. qt. gal. hhd. gal.
(4) 443÷2=221 1 : 221÷4=55 1 : 55÷54=1 1.
hhd. gal. qt. pt.
Ans. 1 1 1 1.

ARTICLE 97.—TIME.

hr. sec.
(1) 2×60×60=7200 *Ans.*

da. min.
(2) 7×24×60=10080 *Ans.*

da. hr. hr. hr. min. min. min. sec. sec.
(3) 1×24+3=27 : 27×60+44=1664 : 1664×60+3=99843 [*Ans.*

wk. da. da. da. hr. hr.
(4) 9×7+6=69 : 69×24+10=1666 :
hr. min. min.
1666×60+40=100000 *Ans.*

mon. wk. wk. da. da. da. hr.
(5) 1×4=4 : 4×7+3=31 : 31×24=744 :
hr. min. min.
744×60+4=44644 *Ans.*

(6) $\overset{\text{yr.}}{1}\times365+\overset{\text{da.}}{20}=\overset{\text{da.}}{385} : \overset{\text{da.}}{385}\times24+\overset{\text{hr.}}{19}=\overset{\text{hr.}}{9259} : \overset{\text{hr.}}{9259}\times60+\overset{\text{min.}}{15}$
$=\overset{\text{min.}}{555555} : \overset{\text{min.}}{555555}\times60+\overset{\text{sec.}}{33}=\overset{\text{sec.}}{33333333}$ *Ans.*

(7) $\overset{\text{da.}}{365}\times24+\overset{\text{hr.}}{5}=\overset{\text{hr.}}{8765} : \overset{\text{hr.}}{8765}\times60+\overset{\text{min.}}{48}=\overset{\text{min.}}{525948} :$
$\overset{\text{min.}}{525948}\times60+\overset{\text{sec.}}{48}=\overset{\text{sec.}}{31556928}$ *Ans.*

(8) $\overset{\text{sec.}}{10800}\div60=\overset{\text{min.}}{180} : \overset{\text{min.}}{180}\div60=\overset{\text{hr.}}{3}$ *Ans.*

(9) $\overset{\text{sec.}}{432000}\div60=\overset{\text{min.}}{7200} : \overset{\text{min.}}{7200}\div60=\overset{\text{hr.}}{120} : \overset{\text{hr.}}{120}\div24=\overset{\text{da.}}{5}$ *Ans.*

(10) $\overset{\text{sec.}}{7322}\div60=\overset{\text{min.}}{122}\ \ \overset{\text{sec.}}{2} : \overset{\text{min.}}{122}\div60=\overset{\text{hr.}}{2}\ \ \overset{\text{min.}}{2}.$ *Ans.* $\overset{\text{hr.}}{2}\ \ \overset{\text{min.}}{2}\ \ \overset{\text{sec.}}{2}.$

(11) $\overset{\text{min.}}{4323}\div60=\overset{\text{hr.}}{72}\ \ \overset{\text{min.}}{3} : \overset{\text{hr.}}{72}\div24=\overset{\text{da.}}{3}.$ *Ans.* $\overset{\text{da.}}{3}\ \ \overset{\text{min.}}{3}.$

(12) $\overset{\text{min.}}{20280}\div60=\overset{\text{hr.}}{338} : \overset{\text{hr.}}{338}\div24=\overset{\text{da.}}{14}\ \overset{\text{hr.}}{2} : \overset{\text{da.}}{14}\div7=\overset{\text{wk.}}{2}.$ *Ans.* $\overset{\text{wk.}}{2}\ \ \overset{\text{hr.}}{2}.$

(13) $\overset{\text{min.}}{41761}\div60=\overset{\text{hr.}}{696}\ \ \overset{\text{min.}}{1} : \overset{\text{hr.}}{696}\div24=\overset{\text{da.}}{29} : \overset{\text{da.}}{29}\div7=\overset{\text{wk.}}{4}\ \ \overset{\text{da.}}{1} :$
$\overset{\text{wk.}}{4}\div4=\overset{\text{mon.}}{1}.$ *Ans.* $\overset{\text{mon.}}{1}\ \ \overset{\text{da.}}{1}\ \ \overset{\text{min.}}{1}.$

ARTICLE 98.—CIRCULAR MEASURE.

(1) $5^\circ\times60+3'=303'$ *Ans.*

(2) $8^\circ\times60+41'=521' : 521'\times60+45''=31305''$ *Ans.*

(3) $3^{s}\times30=90^\circ : 90^\circ\times60+25'=5425'$ *Ans.*

(4) $1^{c}\times360\times60\times60=1296000''$ *Ans.*

(5) $244''\div60=4'\ 4''$ *Ans.* **(6)** $915'\div60=15^\circ\ 15'$ *Ans.*

(7) $1861'\div60=31^\circ\ 1' : 31^\circ\div30=1^{s}\ 1^\circ.$ *Ans.* $1^{s}\ 1^\circ\ 1'.$

ARTICLE 100.

bu. pt. cts. cts. $
(1) 2×4×8×2=128 : 5×128=640=6.40 *Ans.*

bu. pk. pk. cts. cts. $
(2) 3×4+2=14 : 50×14=700=7.00 *Ans.*

pk. qt. qt. qt. pt. cts. $
(3) 3×8+3=27 : 27×2=54 : 3×54=1.62 *Ans.*

$ cts. cts. cts. pk. pk. bu.
(4) 3=300 : 300÷15=20 : 20÷4=5 *Ans.*

$ cts. cts. cts. qt.
(5) 1.66=166 : 166÷4=41 and 2 cts. over, which will buy
qt. pk. qt. pk. bu. pk.
1 pt. at 4 cts. a qt. 41÷8=5 1 : 5÷4=1 1.
bu. pk. qt. pt.
Ans. 1 1 1 1. Or thus: 4 cts. a quart is 2 cts.
cts. cts. pt. bu. pk. qt. pt.
a pt.; and 166÷2=83=1 1 1 1 *Ans.*

bu. pk. pk. bu. pk. pk. pk. bags.
(6) 3 2=14 : 91=364 : 364÷14=26 *Ans.*

lb. pwt. gr. cts. cts. $
(7) 1 3=5832 : 3×5832=17496=174.96 *Ans.*

lb. oz. pwt. pwt. oz. pwt. pwt. pwt. pwt.
(8) 2 5 5=585 : 2 5=45 : 585÷45=13 *Ans.*

lb. oz. pwt. gr. gr. pwt. gr. gr. gr. gr.
(9) 1 8 18 1=10033 : 5 7=127 : 10033÷127=79 *Ans.*

℔ ʒ gr. gr. gr. doses. cts. m. m.
(10) 1 4=6000 : 6000÷15=400 : 12 5=125
m. m. $
125×400÷50000=50 *Ans.*

℔ ℥ ʒ ℈ cts. cts. $
(11) 1 1 1=315 : 4×315=1260=12.60 *Ans.*

cwt. qr. lb. cts. cts. $
(12) 6 1=625 : 625×3=1875=18.75 *Ans.*

T. cwt. cwt. qr. $ cts. $
(13) 1 1=21=84 : 2.25×84=189.00 *Ans.*

lb. oz. oz. cts. cts. $
(14) 7 8=120 : 5×120=600=6 *Ans.*

gr. gr. ℔ ℥ ʒ ℈
(15) 20×316=6320=1 1 1 1 *Ans.*

lb. oz. dr. dr. dr. dr.
(16) 15 9 12=3996 : 3996÷4=999 *Ans.*

cwt. qr. lb. oz. lb. oz. oz. oz. oz.
(17) 44 2 2=71232 : 9 15=159 : 71232÷159=448 *Ans.*

cwt. qr. lb. lb. lb. lb.
(18) 14 1 3=1428 : 1428÷84=17 *Ans.*

cwt. qr. lb. lb. lb. lb.
(19) 7 2 6=756 : 756÷12=63 *Ans.*

cwt. lb. oz. lb. oz. oz. oz. oz.
(20) 6 10=9760 : 3 13=61 : 9760÷61=160 *Ans.*

A. R. P. P. cts. cts. $
(21) 2 3 5=445 : 20×445=8900=89 *Ans.*

sq.yd. sq.ft. sq.ft. sq.in. cts. cts. $
(22) 2 2 = 20=2880 : 2880×5=14400=144 *Ans.*

sq.yd. sq.in. in. in. sq.in. sq.in. sq.in.
(23) 1=1296 : 3×2=6 : 1296÷6=216 *Ans.*

A. R. P. P. A. R. P. P. P. P.
(24) 16 1 13=2613 : 1 1 1=201 : 2613÷201=13 *Ans.*

cu.ft. cu.ft. cu.in.
(25) 2×2×2=8 : 8×1728=13824 *Ans.*

oz. oz. lb. oz.
(26) 1000×5=5000=312 8 *Ans.*

oz. oz. T.
(27) 1000×128=128000=4 *Ans.*

C. cu.ft. oz. oz. T. cwt.
(28) 2×128=256 : 950×256=243200=7 12 *Ans.*

yd. qr. na. na. yd. na. na. na. pieces.
(29) 13 3 2=222 : 666=10656 : 10656÷222=48 *Ans.*

yd. qr. qr. yd. qr. qr. qr. suits.
(30) 5 1=21 : 147=588 : 588÷21=28 *Ans.*

yd. qr. na. na. yd. na. na. na.
(31) 2 1 1=37 : 37=592 : 592÷37=16 *Ans.*

yd. qr. qr. E.Fl. qr. qr. qr. suits.
(32) 3 2=14 : 70=210 : 210÷14=15 *Ans.*

hhd. gi. cts. cts. $
(33) 1=2016 : 5×2016=10080=100.80 *Ans.*

gal. qt. qt. qt. qt. cts. cts. $
(34) 31 2=126 : 126×5=630 : 10×630=6300=63 *Ans.*

$ cts. cts. cts. pt. pt. gal.
(35) 2=200 : 200÷5=40 : 40=5 *Ans.*

$ m. cts. m. m. m. gi. gal.
(36) 12=12000 : 1 5=15 : 12000÷15=800=25 *Ans.*

hhd. pt. qt. pt. pt. pt. pt.
(37) 1=504 : 3 1=7 : 7×12=84 in 1 doz. bottles:
pt. pt. doz.
504÷84=6 *Ans.*

gal. qt. pt. pt. hhd. pt. pt. pt. pt.
(38) 4 3 1=39 : 1=504 : 504÷39=12 and 36 left:
pt. gal. pt. gal. qt.
36=4 2. *Ans.* 12; and 4 2 left.

gal. pt. gal. qt. pt. pt. pt. pt.
(39) 165=1320 1 1 1=11 : 1320÷11=120 *Ans.*

hhd. qt. cts. cts. $
(40) 1=216 : 3×216=648=6.48 *Ans.*

da. min. beats. beats.
(41) 1=1440 : 70×1440=100800 *Ans.*

(42) 1840 is a leap year, because it is exactly divisible by 4; hence, February has 29 days:
29 days=2505600 seconds. *Ans.*

(43) 1839 is not a leap year, not being exactly divisible by 4; hence, February contains 28 days. January has 31 days, which is 3 days more: 3 days=72 hours. *Ans.*

(44) By dividing by 4, it will be found that 1838 and 1839 are not leap years, and that 1840 is one; by adding 4 to this, we find that 1844 and 1848 are both leap years.

da. hr. min. sec. sec. sec. sec.
(45) 365 5 48 48=31556928 : 1609403328÷31556928
yr.
=51 *Ans.*

yr. da. da. min. min. da. hr.
(46) 60×365=21900 : 30×21900=657000=456 6 *Ans.*

wk. da. hr. hr. mi. mi.
(47) 3 2 3=555 : 8×555=4440 *Ans.*

da.
(48) $2^{s}\ 4^{\circ}=64^{\circ}$: $64^{\circ}\div 2^{\circ}=32$ *Ans.*

min.
(49) $90^{\circ}=324000''$: $43'\,12''=2592''$: $324000''\div 2592''=125$
hr. min.
=2 5 *Ans.*

cts. cts. $
(50) 2×20=40 quires : 20×40=800=8 *Ans.*

cts. cts. $
(51) 3×24=72 sheets : 2×72=144=1.44 *Ans.*

doz. cts. cts.
(52) 3=36 apples : 2×36=72 *Ans.*

(53) 1 dozen dozen=12×12=144, and 144×4=576 oranges:
cts. cts. $
3×576=1728=17.28 *Ans.*

(54) 1 gross =12 dozen, and 5 gross =60 dozen:
cts. cts. $
5×60=300=3 *Ans.*

cts. cts. $
(55) 4×196=784=7.84 *Ans.*

m. m. $
(56) 125×200=25000=25 *Ans.*

(57) 1 dozen dozen =144 : 144×6=864 eggs in all : half a dozen dozen =72 : 864—72=792 eggs left :

ct. cts. $
1×792=792=7.92 *Ans.*

NOTE.—In the solutions to several of the preceding examples, it may appear that we have not adhered to the principle of always regarding the multiplier as an abstract number; for instance, in solution to question 22, it might be supposed that we multiply 2880 by 5 cents. In this case, however, the 2880 is to be regarded as the multiplier, and the 5 cents as the multiplicand. It is an established principle, that where two abstract numbers are to be multiplied together, either of them may be made the multiplier. The same principle applies where one of the numbers is concrete: thus, in example 22, the 5 cents may be taken 2880 times, or 2880 cents may be taken 5 times; the result, in either case, being 14400 cents. For convenience, the smaller number is generally made the multiplier.

For the sake of occupying the least possible space with the solutions, we have frequently represented the quotient of one concrete number divided by another, as a concrete number; though, in all cases, it is really abstract, as stated in the remarks on Division, page 19. Thus, in the solution to question 6, preceding, pecks divided by pecks are represented as giving bags for a quotient; the analysis of the question, however, shows, that 14 pecks is contained in 364 pecks 26 times, and as it requires 1 bag to hold 14 pecks, it will require 26 bags to hold 364 pecks; hence, the answer to the question is 26 bags. A similar method of analysis applies to several of the other examples. The quotient *in all cases of division is an abstract number*, and the analysis of the question under consideration always determines what name or denomination is to be given to it.

ADDITION OF COMPOUND NUMBERS.

SUGGESTIONS TO TEACHERS.—The attention of the pupil should be particularly called to the nature of compound numbers, and the analogy that exists between them and simple numbers.

In the latter, the number of each order of units required to form a unit of the next higher order is uniform, ten units in any order always making one unit of the next higher order. In compound numbers the rate of increase is not tenfold, nor is it the same for the same table. Owing to this peculiarity, in compound addition, instead of carrying one for every ten, as in simple addition, we sometimes carry one for every two; again, we carry one for every three, etc. The reason for carrying in compound addition is easily understood by every pupil, and advantage may be taken of this, to explain more clearly the reason for carrying in simple addition.

ANSWERS TO THE QUESTIONS IN COMPOUND ADDITION, TO WHICH THERE ARE NO ANSWERS IN THE BOOK.

ARTICLE 101.

(4) 87 lb. 9 oz. 17 pwt. 22 gr. **(5)** 19 ℔ 11 ℥ 3 ʒ 2 ℈ 1 gr.

(6) 79 T. 15 cwt. 3 qr. 20 lb. **(7)** 94 cwt. 3 qr. 5 lb. 5 oz. 5 dr.

(8) 38 mi. 5 fur. 19 rd. **(9)** 12 yd. 4 in.

(10) 299 A. 3 R. 30 P. **(11)** 51 sq. yd. 4 sq. ft. 73 sq. in.

(12) 49 C. 58 cu. ft. 519 cu. in.

(13) 143 cu. yd. 2 cu. ft. 990 cu. in.

(14) 26 yd. 3 qr. 2 na. **(15)** 37 E. Fl. 2 qr. 3 na.

(16) 46 E. En. 2 na.

(17) 26 T. 34 gal. 2 qt. **(18)** 128 gal. 3 qt. 1 pt. 3 gi.

(19) 8 mon. 4 da. 8 hr. 49 min. 35 sec. **(20)** 329° 5′ 52″

(21)	bu.	pk.	qt.	pt.
	21	3	2	1
	14	0	5	0
	23	2	0	1
	18	1	0	1
	22	0	7	1
Ans.	100	0	0	0

(22)	bu.	pk.	qt.	pt.
	200	3	0	1
	143	0	2	1
	400	3	0	0
	255	1	5	0
Ans.	1000	0	0	0

(23)	lb.	oz.	pwt.	gr.
	2	10	15	21
	1	1	16	14
	2	8	5	12
Ans.	6	8	17	23

(24)	℔	℥	ʒ	℈	gr.
		3	4	1	0
		4	3	2	0
			4	0	18
		6	5	2	18
Ans.	1	3	2	0	16

(25)	cwt.	qr.	lb.
	8	1	11
	4	2	14
	5	0	19
	7	3	0
	7	3	9
Ans.	33	3	3

(26)	lb.	oz.	dr.
	13	11	15
	17	13	11
	14	14	0
	16	0	10
	19	7	12
	17	9	9
Ans.	99	9	9

(27)	mi.	fur.	rd.
	104	1	10
	95	6	30
Ans.	200	0	0

(28)	A.	R.	P.
	186	3	14
	286	0	17
	113	2	9
Ans.	586	2	0

(29)	sq. yd.	sq. ft.	sq. in.
	17	3	119
	18	0	141
	23	7	00
	29	5	116
Ans.	88	8	88

(30)	C.	cu.ft.
	7	78
	16	24
	35	127
	29	10
Ans.	88	111

(31)	yd.	qr.	na.
	17	3	2
	13	2	1
	23	0	2
	27	1	2
	29	1	2
Ans.	111	1	1

(32)	hhd.	gal.	qt.	pt.
	73	43	3	1
	27	3	0	0
	15	0	3	1
	162	0	1	0
Ans.	277	48	0	0

(33)	bl.	gal.	qt.	pt.
	1	28	0	0
	1	17	0	0
	5	2	0	0
		1	2	1
		7	2	1
		18	3	0
		33	0	0
Ans.	10	0	0	0

(34)		
	Jan.	31
	Feb.	28
	March	31
	April	30
	May	1
	Ans.	121

(35)		
	Jan.	31
	Feb.	29
	March	31
	April	30
	May	31
	June	30
	July	4
	Ans.	186

(36)					
	Cape Horn,	55°	58′	30″	S.
	New York,	40	42	40	N.
	Ans.	96	41	10	

SUBTRACTION.

Suggestions to Teachers.—After the pupil clearly understands the method of borrowing when the lower denomination is larger than the upper, his attention should be directed to the method of borrowing in simple subtraction, as he will then be better prepared to understand the rule there given.

ANSWERS TO QUESTIONS IN COMPOUND SUBTRACTION, TO WHICH THERE ARE NO ANSWERS IN THE BOOK

Article 102.

(4) 8 lb. 11 oz. 18 pwt. 7 gr. **(5)** 4 T. 2 cwt. 3 qr. 5 lb. 15 oz. 3 dr.

(6) 7 mi. 38 rd. **(7)** 1 yd. 2 ft. 11 in.

(8) 250 A. 38 P. **(9)** 2 sq. yd. 8 sq. ft. 104 sq. in.

(10) 55 da. 5 hr. 55 min. 55 sec. **(11)** 9° 59′ 56″

	bu.	pk.	qt.
(12)	4	0	0
	2	1	1
Ans.	1	2	7

	bu.	pk.	qt.	pt.
(13)	100	0	0	0
	24	0	0	1
Ans.	75	3	7	1

	lb.	oz.	pwt.	gr.
(14)	19	9	0	0
	9	9	10	10
Ans.	9	11	9	14

	lb.	oz.	pwt.	gr.
(15)	8	2	11	0
	2	4	7	16
Ans.	5	10	3	8

	℔	℥	ʒ	℈	gr.
(16)	3	3	1	1	12
	1	7	0	2	18
Ans.	1	8	0	1	14

	lb.	oz.	dr.
(17)	46	9	0
	19	0	4
Ans.	27	8	12

	cwt.	qr.	lb.
(18)	32	2	16
	8	2	17
Ans.	23	3	24

	mi.	fur.	rd.
(19)	25000	0	0
	100	1	1
Ans.	24899	6	39

	A.	R.	P.
(20)	146	2	0
	86	2	14
Ans.	59	3	26

	C.	cu. ft.
(21)	8	50
	3	75
Ans.	4	103

	E.Fl.	qr.	na.
(22)	25	0	0
	14	1	3
Ans.	10	1	1

	yd.	qr.	na.
(23)	11	0	0
	3	2	2
Ans.	7	1	2

	gal.	qt.	pt.	gi.
(24)	63	0	0	0
	51	1	0	2
Ans.	11	2	1	2

	da.	hr.	min.	sec.
(25)	5	10	27	15
	2	4	13	29
Ans.	3	6	13	46

	yr.	mon.	da.
(27)	1857	3	5
	1776	7	4
Ans.	80	8	1

	yr.	mon.	da.
(28)	1848	8	1
	1822	6	24
Ans.	26	1	7

	yr.	mon.	da.
(29)	1832	6	28
	1807	11	25
Ans.	24	7	3

(30)	55°	58′	30″
	33	55	15
Ans.	22	3	15

(31)	71°	10′	0″
	36	6	30
Ans.	35	3	30

(32)	50°	0′	0″
	10	25	48
Ans.	39	34	12

	quires.	sheets.
(33)	20	0
	5	11
Ans.	14	13

(34) 1 dozen dozen = 12×12=144
6 dozen dozen =144× 6=864
$\frac{1}{2}$ a dozen dozen =144÷ 2= 72
Ans. 792

ARTICLE 104.—MULTIPLICATION.

	bu.	pk.	qt.	pt.
(4)	4	3	3	1
				12
Ans.	58	1	2	0

	lb.	oz.	pwt.	gr.
(5)		2	11	6
				6
	1	3	7	12

	lb.	oz.	pwt.	gr.
(6)		10	10	10
				10
	8	9	4	4

	℔	℥	ʒ	℈	gr.
(7)		4	0	2	15
					8
Ans.	2	8	7	1	0

	T.	cwt.	qr.	lb.
(8)		8	2	12
				9
Ans.	3	17	2	8

	T.	cwt.	qr.	lb.
(9)		10	3	14
				7
Ans.	3	16	0	23

	mi.	fur.	rd.
(10)	208	4	16
			15
Ans.	3128	2	0

	mi.	fur.	rd.
(11)	30	4	10
			12
Ans.	366	3	0

	A.	R.	P.
(12)	130	3	30
			4
Ans.	523	3	0

	cu.yd.	cu.ft.	cu.in.
(13)	23	9	228
			12
Ans.	280	1	1008

	yd.	qr.	na.
(14)	26	2	2
			6
Ans.	159	3	0

	gal.	qt.	pt.
(15)	62	1	1
			8
Ans.	499	0	0

	gal.	qt.	pt.
(16)	123	2	1
			5
Ans.	618	0	1

	da.	hr.	min.	sec.
(17)	365	5	48	48
				12
Ans.	4382	21	45	36

(18)^{s}		4°	11′	15″
				8
Ans.	1	3	30	0

	doz.
(19)	144
	12
	1728
	3
Ans.	5184

	bu.	pk.	qt.
(21)	2	3	5
			4
	11	2	4
			6
Ans.	69	3	0

	mi.	fur.	rd.
(22)	3	5	16
			6
	22	0	16
			10
Ans.	220	4	0

	A.	R.	P.
(23)	6	3	30
			8
	55	2	0
			7
Ans.	388	2	0

	T.	cwt.	qr.	lb.	oz.	dr.
(24)		8	2	14	12	13
						11
		95	0	12	12	15
						2
Ans.	9	10	1	0	9	14

	hhd.	gal.	qt.	pt.	gi.
(25)		3	2	1	1
					8
		29	1	0	0
					2
		58	2	0	0
					7
Ans.	6	31	2	0	0

(26)

	T.	cwt.	qr.	lb.
		16	2	24
				119
Ans.	99	12	0	6

(27)

	yd.	qr.	na.
	37	3	2
			89
Ans.	3370	3	2

(28)

	hhd.	gal.	qt.	pt.
		47	3	1
				59
Ans.	44	52	2	1

(29)

	mi.	fur.	rd.
	27	3	35
			31
Ans.	852	0	5

(30)

	C.	cu. ft.
	7	98
		17
Ans.	132	2

ARTICLE 106.—DIVISION.

(4)

	bu.	pk.	qt.	pt.
	5)67	3	4	1
Ans.	13	2	2	1

(5)

	lb.	oz.	pwt.	gr.
	8)14	8	16	16
Ans.	1	10	2	2

(6)

	oz.	pwt.	gr.
	10)8	12	12
Ans.		17	6

(7)

	cwt.	qr.	lb.	oz.	dr.
	11)35	1	17	3	7
Ans.	3	0	22	0	5

(8)

	mi.	fur.	rd.
	7)39	7	8
Ans.	5	5	24

(9)

	yd.	qr.	na.
	5)62	0	3
Ans.	12	1	3

(11)

	bu.	pk.	qt.
	10)490	2	4
	10)49	0	2
Ans.	4	3	5

(12)

	lb.	oz.	dr.
	5)266	9	10
	10)53	5	2
Ans.	5	5	5

(13)

	lb.	oz.	pwt.	gr.
	6)339	7	9	18
	7)56	7	4	23
Ans.	8	1	0	17

(14)

	da.	hr.	min.	sec.	
	6)114	22	45	35	
	9)19	3	47	35	5 rem.
Ans.	2	3	5	17	2 rem.

2×6+5=17 sec. rem.

(16)

```
   lb. oz. dr. lb. oz. dr.
23)1027  1  8(44  10  8 Ans.
   92
   ---
   107
    92
    --
    15
    16
    --
    91
   15
   ---
23)241(10 oz.
   23
   ---
    11
    16
    --
    74
   11
   ---
23)184(8 dr.
   184
   ---
```

(17)

```
    lb. oz. pwt. gr. lb.oz.pwt.gr.
245)17   7   6   6 ( 0  0  17  6
    12                      [Ans.
    --
245)211(0 oz.
     20
    ---
245)4226(17 pwt
    245
    ----
    1776
    1715
    ----
      61
      24
    ----
     250
    122
    ----
245)1470(6 gr.
    1470
    ----
```

(18)

```
   bu.  pk. qt. bu. pk. qt.
78)309   2   2( 3   3   7 Ans.
   234
   ---
    75
     4
   ---
78)302(3 pk.
   234
   ---
    68
     8
   ---
78)546(7 qt.
   546
   ---
```

(19)

```
    mi.  fur. rd. mi. fur. rd.
319)788   4   9( 2   3   31 Ans.
    638
    ---
    150
      8
    ----
319)1204(3 fur.
     957
    ----
     247
      40
    ----
319)9889(31 rd.
    957
    ----
     319
     319
    ----
```

REMARK.—For cases involving the division of a compound number by a fraction; also, one compound number by another, see "*Ray's Higher Arithmetic.*"

ARTICLE 107.

(1)

	℔	℥	ʒ	℈	gr.
	1	0	0	0	0
		4	2	0	13
		7	5	2	7
		3	1	2	14
Ans.		4	3	2	13

Or, add the two quantities together, and take their sum from 1 ℔.

(2)

	oz.	pwt.	gr.
	8	10	12
	9	3	5
	8	9	7
	26	3	0
		5	12
	3)25	17	12
	5)8	12	12
Ans.	1	14	12

(3)

	A.	R.	P.
	104	2	37
	87	1	38
	192	0	35
	40	1	0
	3)151	3	35
Ans.	50	2	25

(4)

	mi.	fur.	rd.
		3	25
			4
	1	6	20
			30
Ans.	54	3	0

(5)

	yr.	mon.	da.
	1845	2	3
	1843	6	27
	1	7	6
			5
Ans.	8	0	0

(6)

	35°	30′	0″
Sub.	20	35	0
	14	55	0
Add	14	20	0
	29	15	0
Sub.	25	4	30
	4	10	30
Add	6	19	20
Ans.	10	29	50

ARTICLE 108.

(1) 15)18° 25′ 30″

Ans. 1 hr. 13 min. 42 sec.

(2) 30° ÷ 15 = 2. *Ans.* 2 hr.

(3) 15)71° 4′ 0″

Ans. 4 hr. 44 min. 16 sec.

(4) 15)10° 35′ 0″

Ans. 0 hr. 42 min. 20 sec.

	min.	sec.
(5)	37	20
		15

Ans. 9° 20′ 0″

	hr.	min.	sec.
(6)	1	4	56
			15

Ans. 16° 14′ 0″

	hr.	min	sec.
(7)	5	8	4
			15

Ans. 77° 1′ 0″

ARTICLE 109.

(8)

	hr.	min.	sec.
Time at C.	12	0	0
Add diff.		37	20
Ans.	12	37	20

(9)

	hr.	min.	sec.	
Time at N. Y.	11	0	0	A. M.
30°=	2	0	0	to be added.
Ans.	1	0	0	P. M.

(10)

	hr.	min.	sec.	
Time at Ph.	12	0	0	
Subtr. diff.		37	20	
Ans.	11	22	40	A. M.

(11)

	hr.	min.	sec.	
Time at N. Y.	11	0	0	
Subtract diff.	1	4	56	
Ans.	9	55	4	A. M.

(12) 124°—80° 42′=43° 18′ : 43° 18′÷15= 2 hr. 53 min. 12 sec.

	hr.	min.	sec.	
Time at W.	1	0	0	
Subtr. diff.	2	53	12	
Ans.	10	6	48	A. M.

NOTE.—In performing the subtraction we can not take 3 hr. from 1 hr., but 1 P. M. is the 13th hour from midnight, from which, after taking 3 hr., the remainder is the 10th hour from midnight, or 10 A. M.

ARTICLE 113.—FACTORING.

NOTE.—Most pupils in a short time may be taught to give the prime factors of all the composite numbers, from 4 to 100, mentally, or by inspection. The great simplicity of the first 21 examples, renders it unnecessary to present their solution here.

(22) 105÷3=35 : 35÷5=7. *Ans.* 3, 5, 7.

(23) 168÷2=84 : 84÷2=42 : 42÷2=21 : 21÷3=7. *Ans.* 2, 2, 2, 3, 7.

(24) 216÷2=108 : 108÷2=54 : 54÷2=27 : 27÷3=9 : 9÷3=3. *Ans.* 2, 2, 2, 3, 3, 3.

(25) 330÷2=165 : 165÷3=55 : 55÷5=11. *Ans.* 2, 3, 5, 11.

(26) 110=2×5×11 : 210=2×3×5×7. *Ans.* 2, 5.

(27) 105=3×5×7 : 231=3×7×11. *Ans.* 3, 7.

(28) 330=2×3×5×11 : 390=2×3×5×13. *Ans.* 2, 3, 5.

(29) 231=3×7×11 : 330=2×3×5×11. *Ans.* 3, 11.

ARTICLE 114.

(1) 42=2×3×7 : hence the divisors are, 2, 3, 7; and 2×3=6; 2×7=14; and 3×7=21. *Ans.*

(2) 105=3×5×7 : hence the divisors are, 3, 5, 7; and 3×5=15; 3×7=21; and 5×7=35. *Ans.*

(3) 20=2×2×5 : hence the divisors are, 2, 5; and 2×2=4; and 2×5=10. *Ans.*

(4) 24=2×2×2×3 : hence the divisors are, 2, 3; and 2×2=4; 2×3=6; 2×2×2=8; and 2×2×3=12. *Ans.*

ARTICLE 116.—RULE I.

(4) 42=2×3×7 : 54=2×3×3×3 : hence 2×3=6 gr. c. d.

(5) 70=2×5×7 : 110=2×5×11 : hence 2×5=10 gr. c. d.

(6) 105=3×5×7 : 165=3×5×11 : hence 3×5=15 g. c. d.

(7) 60=2×2×3×5 : 90=2×3×3×5 : hence 2×3×5 =30 g. c. d.

(8) 140=2×2×5×7 : 210=2×3×5×7 : hence 2×5×7 =70 g. c. d.

(9) 66=2×3×11 : 154=2×7×11 : hence 2×11=22 g. c. d.

(10) 154=2×7×11 : 280=2×2×2×5×7 : hence 2×7 =14 g. c d.

(11) 231=3×7×11 : 273=3×7×13 : hence 3×7=21 g. c. d.

(12) 30=2×3×5 : 42=2×3×7 : 66=2×3×11 : hence 2×3=6 g. c. d.

(13) 60=2×2×3×5 : 90=2×3×3×5 : 150=2×3×5×5 : hence 2×3×5=30 g. c. d.

ARTICLE 117.—RULE II.

(4) 495÷42=11+33 r. : 42÷33=1+9 r. : 33÷9=3+6 r. : 9÷6=1+3 r. : 6÷3=2. *Ans.* 3.

(5) 323÷247=1+76 r. : 247÷76=3+19 r. : 76÷19=4. *Ans.* 19.

(6) 465÷285=1+180 r. : 285÷180=1+105 r. : 180÷105 =1+75 r. : 105÷75=1+30 r. : 75÷30=2+15 r : 30÷15=2. *Ans.* 15.

(7) $1274\div532=2+210$ r. : $532\div210=2+112$ r. : $210\div112=1+98$ r. : $112\div98=1+14$ r. : $98\div14=7$; *Ans.* 14.

(8) $759\div337=2+85$ r. : $337\div85=3+82$ r. : $85\div82=1+3$ r. : $82\div3=27+1$ r. : $3\div1=3$. *Ans.* 1.

(9) $69087\div9873=6+9849$ r. : $9873\div9849=1+24$ r. : $9849\div24=410+9$ r. : $24\div9=2+6$ r. : $9\div6=1+3$ r. : $6\div3=2$. *Ans.* 3.

(10) $1814\div1776=1+38$ r. : $1776\div38=46+28$ r. : $38\div28=1+10$ r. : $28\div10=2+8$ r. : $10\div8=1+2$ r. : $8\div2=4$. *Ans.* 2.

(11) $1815\div693=2+429$ r. : $693\div429=1+264$ r. : $429+264=1+165$ r. : $264\div165=1+99$ r. : $165\div99=1+66$ r. : $99\div66=1+33$ r. : $66\div33=2$. *Ans.* 33.

(12) $3471\div2145=1+1326$ r. : $2145\div1326=1+819$ r. : $1326\div819=1+507$ r. : $819\div507=1+312$ r. : $507\div312=1+195$ r. : $312\div195=1+117$ r. : $195\div117=1+78$ r. : $117\div78=1+39$ r. : $78\div39=2$. *Ans.* 39.

(13) $17017\div840=20+217$ r. : $840\div217=3+189$ r. : $217\div189=1+28$ r. : $189\div28=6+21$ r. : $28\div21=1+7$ r. : $21\div7=3$. *Ans.* 7.

(14) $153452\div66284=2+20884$ r. : $66284\div20884=3+3632$ r. : $20884\div3632=5+2724$ r. : $3632\div2724=1+908$ r. : $2724\div908=3$. *Ans.* 908.

(15) $55\div40=1+15$ r. : $40\div15=2+10$ r. : $15\div10=1+5$ r. : $10\div5=2$. Again, $105\div5=21$. *Ans.* 5. Or, by resolving the numbers into their prime factors, and taking the product of those factors that are common. $40=2\times2\times2\times5$. $55=5\times11$. $105=3\times7\times5$. Since 5 is the only factor common to each of the numbers, it is their only common divisor.

(16) $154\div70=2+14$ r. : $70\div14=5$. Again, $819\div14=58+7$ r. : $14\div7=2$. *Ans.* 7.

(17) $168\div120=1+48$ r. : $120\div48=2+24$ r. : $48\div24=2$. Again, $1768\div24=73+16$ r. : $24\div16=1+8$ r. : $16\div8=2$. *Ans.* 8. Or, $120=2\times2\times2\times3\times5$. $168=2\times2\times2\times3\times7$. $1768=2\times2\times2\times13\times17$, hence, $2\times2\times2=8$, is the greatest common divisor.

ARTICLE 119.—RULE I.

(3) $12=2\times2\times3$
$30=\cancel{2}\times\cancel{3}\times5$
$2\times2\times3\times5=60$ *Ans.*

(4) $6=2\times3$
$10=\cancel{2}\times5$
$18=\cancel{2}\times\cancel{3}\times3$
$2\times3\times5\times3=90$ *Ans.*

(5) $6=2\times3$
$8=\cancel{2}\times2\times2$
$9=\cancel{3}\times3$
$2\times3\times2\times2\times3=72$ *Ans.*

(6) $6=2\times3$
$15=\cancel{3}\times5$
$35=\cancel{5}\times7$
$2\times3\times5\times7=210$ *Ans.*

(7) $10=2\times5$
$12=\cancel{2}\times2\times3$
$15=\cancel{3}\times\cancel{5}$
$2\times5\times2\times3=60$ *Ans.*

(8) $9=3\times3$
$15=\cancel{3}\times5$
$18=2\times\cancel{3}\times\cancel{3}$
$24=\cancel{2}\times2\times2\times\cancel{3}$
$3\times3\times5\times2\times2\times2=360$ *Ans.*

(9) $8=2\times2\times2$
$15=3\times5$
$12=\cancel{2}\times\cancel{2}\times\cancel{3}$
$30=\cancel{2}\times\cancel{3}\times\cancel{5}$
$2\times2\times2\times3\times5=120$ *Ans.*

(10) $14=2\times7$
$21=3\times\cancel{7}$
$30=\cancel{2}\times\cancel{3}\times5$
$35=\cancel{5}\times\cancel{7}$
$2\times7\times3\times5=210$ *Ans.*

REMARK.—When the numbers are small, or when they are few, Rule 1st is preferable; but in other cases Rule 2d is the best. It will be advantageous to require the pupil to work several of the same examples by both rules.

ARTICLE 120.—RULE II.

(2) 2)6 10
3 5
$2\times3\times5=30$ *Ans.*

(3) 3)15 21 35
5)5 7 35
7)1 7 7
1 1 1
$3\times5\times7=105$ *Ans.*

(4) 3)9 12
3 4
$3\times3\times4=36$ *Ans.*

(5) 7)14 21
2 3
$7\times2\times3=42$ *Ans.*

(6) 3)6 9 15
2 3 5
$3\times2\times3\times5=90$ *Ans.*

(7) 2)4 14 35
7)2 7 35
2 1 5
$2\times7\times2\times5=140$ *Ans.*

(8)
$$\begin{array}{rrrr} 2)6 & 10 & 15 & 18 \\ \hline 3)3 & 5 & 15 & 9 \\ \hline 5)1 & 5 & 5 & 3 \\ \hline 1 & 1 & 1 & 3 \end{array}$$
2×3×5×3=90 *Ans.*

(9) These numbers contain no common factor, therefore their least common multiple is their product.

7×11×13×3=3003 *Ans.*

(10)
$$\begin{array}{rrrr} 2)63 & 12 & 84 & 7 \\ \hline 2)63 & 6 & 42 & 7 \\ \hline 3)63 & 3 & 21 & 7 \\ \hline 7)21 & 1 & 7 & 7 \\ \hline 3 & 1 & 1 & 1 \end{array}$$
2×2×3×7×3=252 *Ans.*

(11)
$$\begin{array}{rrr} 3)54 & 81 & 63 \\ \hline 3)18 & 27 & 21 \\ \hline 3)6 & 9 & 7 \\ \hline 2 & 3 & 7 \end{array}$$
3×3×3×2×3×7=1134 *Ans.*

(12)
$$\begin{array}{rrrrr} 2)8 & 12 & 20 & 24 & 25 \\ \hline 2)4 & 6 & 10 & 12 & 25 \\ \hline 2)2 & 3 & 5 & 6 & 25 \\ \hline 3)1 & 3 & 5 & 3 & 25 \\ \hline 5)1 & 1 & 5 & 1 & 25 \\ \hline 1 & 1 & 1 & 1 & 5 \end{array}$$
2×2×2×3×5×5=600 *Ans.*

(13)
$$\begin{array}{rrrrrr} 2)9 & 10 & 24 & 25 & 32 & 45 \\ \hline 2)9 & 5 & 12 & 25 & 16 & 45 \\ \hline 2)9 & 5 & 6 & 25 & 8 & 45 \\ \hline 3)9 & 5 & 3 & 25 & 4 & 45 \\ \hline 3)3 & 5 & 1 & 25 & 4 & 15 \\ \hline 5)1 & 5 & 1 & 25 & 4 & 5 \\ \hline 1 & 1 & 1 & 5 & 4 & 1 \end{array}$$
2×2×2×3×3×5×5×4
=7200 *Ans.*

(14)
$$\begin{array}{rrrrr} 2)98 & 72 & 64 & 21 & 18 \\ \hline 2)49 & 36 & 32 & 21 & 9 \\ \hline 2)49 & 18 & 16 & 21 & 9 \\ \hline 3)49 & 9 & 8 & 21 & 9 \\ \hline 3)49 & 3 & 8 & 7 & 3 \\ \hline 7)49 & 1 & 8 & 7 & 1 \\ \hline 7 & 1 & 8 & 1 & 1 \end{array}$$
2×2×2×3×3×7×7×8=28224 *Ans.*

(15)
$$\begin{array}{rrrrrrrr} 2)2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ \hline 2)1 & 3 & 2 & 5 & 3 & 7 & 4 & 9 \\ \hline 3)1 & 3 & 1 & 5 & 3 & 7 & 2 & 9 \\ \hline 1 & 1 & 1 & 5 & 1 & 7 & 2 & 3 \end{array}$$
2×2×3×5×7×2×3=2520 *Ans.*

COMMON FRACTIONS.

Suggestions to Teachers.—A thorough knowledge of fractions is indispensable to a proper understanding of all the most important and useful parts of Arithmetic, and is also of great utility in almost every branch of science; therefore, no pains should be spared to render the pupil perfectly acquainted with the subject.

Previous to commencing fractions in written arithmetic, it is desirable that the pupil should have studied Mental Fractions. (See Ray's Arithmetic, 2d Book, page 86, following.) When this has not been attended to, the instructor will find it advantageous, at the commencement of each case, to prepare questions to be solved mentally, similar to those which follow.

As far as it may be practicable, let the pupil be made to understand clearly the nature of every operation he is required to perform. In doing this, the examples and illustrations derived from mental arithmetic will be found of great value.

Article 138.—Case I.

Note.—For mental exercises, and the method of illustrating the subject, see Ray's Arithmetic, 2d Book, page 95.

The only difficulty in solving the examples in this section, consists in finding the greatest common divisor of both terms of the fraction. The following are the greatest common divisors.

(2) 6. **(3)** 30. **(4)** 6. **(5)** 15. **(6)** 30. **(7)** 14.
(8) 16. **(9)** 5. **(10)** 18. **(11)** 14. **(12)** 15. **(13)** 97.
(14) 111. **(15)** 101. **(16)** 23. **(17)** 117. **(18)** 199.

Article 139.—Case II.

From the 1st and 2d questions in this Article, it will be readily seen that the reduction of an improper fraction to a whole or mixed number, involves the same principle as reduction from a lower to a higher denomination. Thus, to reduce 17 pecks to bushels, and to reduce 17 fourths to units, or $\frac{17}{4}$ to a mixed number, all involve the same principle, and are performed in the same manner.

NOTE.—For mental exercises embracing the principles of this case, see Ray's Arithmetic, 2d Book, page 90.

The examples in this Article being all performed by division, the exhibition of the manner of doing the work would consist simply in writing down the question and then the answer; thus,

Q. (6), $\frac{25}{3}=8\frac{1}{3}$.

ARTICLE 140.—CASE III.

From the first six examples it will be seen that the reduction of a whole or mixed number to an improper fraction, involves the same principle as the reduction from a higher denomination to a lower.

NOTE.—For mental exercises embracing the principles of this case, see Ray's Arithmetic, 2d book, page 89.

(7) $8\times4+3=35$. *Ans.* $\frac{35}{4}$. (8) $12\times5+3=63$. *Ans.* $\frac{63}{5}$
(9) $5\times10+3=53$. *Ans.* $\frac{53}{10}$. (10) $15\times6+3=93$. *Ans.* $\frac{93}{6}$.
(11) $26\times24+13=637$. *Ans.* $\frac{637}{24}$.
(12) $8\times2+1=17$. *Ans.* $\frac{17}{2}$. (13) $5\times4+1=21$. *Ans.* $\frac{21}{4}$.
(14) $3\times55+17=182$. *Ans.* $\frac{182}{55}$.
(15) $46\times8+5=373$. *Ans.* $\frac{373}{8}$.
(16) $21\times583+117=12360$. *Ans.* $\frac{12360}{583}$.
(17) $1\times1000+999=1999$. *Ans.* $\frac{1999}{1000}$.
(18) $14\times71+6=1000$. *Ans.* $\frac{1000}{71}$.
(19) $10\times111+1=1111$. *Ans.* $\frac{1111}{111}$.

ARTICLE 141.

(2) $4=\frac{4\times7}{1\times7}=\frac{28}{7}$ *Ans.* (3) $8=\frac{8\times9}{1\times9}=\frac{72}{9}$ *Ans.*

(4) $19=\frac{19\times19}{1\times19}=\frac{361}{19}$ *Ans.* (5) $37=\frac{37\times23}{1\times23}=\frac{851}{23}$ *Ans.*

ARTICLE 142.—CASE IV.

(5) $\frac{7}{11}\times\frac{2}{3}=\frac{14}{33}$ *Ans.* **(6)** $\frac{3}{4}\times\frac{5}{8}=\frac{15}{32}$ *Ans.*

(7) $1\frac{4}{9}=\frac{13}{9}$; then, $\frac{2}{3}+\frac{5}{7}\times\frac{13}{9}=\frac{130}{189}$ *Ans.*

ARTICLE 143.

(12) $\frac{1}{\cancel{3}}\times\frac{\cancel{3}}{\cancel{4}}\times\frac{\cancel{4}}{9}=\frac{1}{9}$ *Ans.* **(13)** $\frac{1}{9}\times\frac{\cancel{3}}{\cancel{4}}\times\frac{\cancel{4}}{\cancel{3}}=\frac{1}{9}$ *Ans.*

(14) $1\frac{17}{18}=\frac{35}{18}$. $\frac{\cancel{3}}{\cancel{5}}\times\frac{\cancel{6}}{\cancel{7}}\times\frac{\cancel{35}}{\cancel{18}}=1$ *Ans.*

(15) $1\frac{3}{4}=\frac{7}{4}$. $\frac{\cancel{3}}{\cancel{7}}\times\frac{8}{\cancel{3}}\times\frac{\cancel{7}}{4}=\frac{8}{4}=2$ *Ans.*

(16) $1\frac{6}{7}=\frac{13}{7}$. $\frac{\cancel{9}}{\cancel{13}}\times\frac{\cancel{7}}{\underset{2}{\cancel{18}}}\times\frac{\cancel{13}}{\cancel{7}}=\frac{1}{2}$ *Ans.*

(17) $\frac{1}{2}\times\frac{\cancel{4}}{\cancel{5}}\times\frac{1}{\underset{2}{\cancel{8}}}\times\frac{\cancel{5}}{1}=\frac{1}{4}$ *Ans.*

(18) $\frac{1}{\cancel{2}}\times\frac{\cancel{2}}{\cancel{3}}\times\frac{\cancel{3}}{\cancel{4}}\times\frac{\cancel{4}}{\cancel{5}}\times\frac{\cancel{5}}{8}\times\frac{\cancel{5}}{\cancel{9}}\times\frac{\cancel{9}}{\underset{2}{\cancel{10}}}=\frac{1}{16}$ *Ans.*

ARTICLE 144.—CASE V.

(5) $\frac{1\times4\times5}{2\times4\times5}=\frac{20}{40}$ **(6)** $\frac{1\times3\times4}{2\times3\times4}=\frac{12}{24}$ **(7)** $\frac{1\times5\times7}{2\times5\times7}=\frac{35}{70}$

$\frac{3\times2\times5}{4\times2\times5}=\frac{30}{40}$ $\frac{1\times2\times4}{3\times2\times4}=\frac{8}{24}$ $\frac{3\times2\times7}{5\times2\times7}=\frac{42}{70}$

$\frac{4\times2\times4}{5\times2\times4}=\frac{32}{40}$ $\frac{1\times2\times3}{4\times2\times3}=\frac{6}{24}$ $\frac{6\times2\times5}{7\times2\times5}=\frac{60}{70}$

(8) $$\frac{1\times3\times4\times5}{2\times3\times4\times5}=\frac{60}{120}$$

$$\frac{2\times2\times4\times5}{3\times2\times4\times5}=\frac{80}{120}$$

$$\frac{3\times2\times3\times5}{4\times2\times3\times5}=\frac{90}{120}$$

$$\frac{4\times2\times3\times4}{5\times2\times3\times4}=\frac{96}{120}$$

(9) $$\frac{2\times5\times7\times8}{3\times5\times7\times8}=\frac{560}{840}$$

$$\frac{2\times3\times7\times8}{5\times3\times7\times8}=\frac{336}{840}$$

$$\frac{3\times3\times5\times8}{7\times3\times5\times8}=\frac{360}{840}$$

$$\frac{5\times3\times5\times7}{8\times3\times5\times7}=\frac{525}{840}$$

(10) $$\frac{6\times7\times13}{11\times7\times13}=\frac{546}{1001}$$

$$\frac{4\times11\times13}{7\times11\times13}=\frac{572}{1001}$$

$$\frac{8\times11\times7}{13\times11\times7}=\frac{616}{1001}$$

(11) $\frac{1}{2}$ of $\frac{3}{4}=\frac{3}{8}$. $2\frac{1}{3}=\frac{7}{3}$. $3=\frac{3}{1}$.

$$\frac{3\times3\times1}{8\times3\times1}=\frac{9}{24}$$

$$\frac{7\times8\times1}{3\times8\times1}=\frac{56}{24}$$

$$\frac{3\times8\times3}{1\times8\times3}=\frac{72}{24}.$$

(12) $\frac{3}{4}$ of $\frac{5}{6}=\frac{5}{8}$. $\frac{1}{2}\times\frac{2}{5}\times\frac{3}{7}\times\frac{5}{3}=\frac{1}{7}$.

$$\frac{5\times7}{8\times7}=\frac{35}{56}$$

$$\frac{1\times8}{7\times8}=\frac{8}{56}$$

ARTICLE 145.

In solution of questions in this Article, the multiplier of both terms of each fraction is supposed to be obtained *mentally*, by dividing the largest denominator by the denominator of the fraction to be reduced.

(2) $$\frac{1\times2}{2\times2}=\frac{2}{4}$$

$$\frac{3}{4}=\frac{3}{4}$$

(3) $$\frac{1\times4}{2\times4}=\frac{4}{8}$$

$$\frac{1\times2}{4\times2}=\frac{2}{8}$$

$$\frac{1}{8}=\frac{1}{8}$$

(4) $$\frac{1\times2}{3\times2}=\frac{2}{6}$$

$$\frac{1\times3}{2\times3}=\frac{3}{6}$$

$$\frac{5}{6}=\frac{5}{6}$$

(5) $$\frac{1\times5}{2\times5}=\frac{5}{10}$$

$$\frac{2\times2}{5\times2}=\frac{4}{10}$$

$$\frac{3}{10}=\frac{3}{10}$$

(6) $\frac{1\times4}{3\times4}=\frac{4}{12}$ **(7)** $\frac{1\times7}{2\times7}=\frac{7}{14}$ **(8)** $\frac{3\times4}{4\times4}=\frac{12}{16}$ **(9)** $\frac{2\times4}{3\times4}=\frac{8}{12}$

$\frac{5\times2}{6\times2}=\frac{10}{12}$ $\frac{4\times2}{7\times2}=\frac{8}{14}$ $\frac{5\times2}{8\times2}=\frac{10}{16}$ $\frac{3\times3}{4\times3}=\frac{9}{12}$

$\frac{7}{12}=\frac{7}{12}$ $\frac{9}{14}=\frac{9}{14}$ $\frac{11}{16}=\frac{11}{16}$ $\frac{11}{12}=\frac{11}{12}$

ARTICLE 146.—CASE VI.

(4) Least common multiple of 2, 3, and 4=12. (Art. 119, 120.)
$\frac{12}{2}=6$, $\frac{12}{3}=4$, $\frac{12}{4}=3$;
$1\times6=6$, $1\times4=4$, $3\times1=3$. *Ans.* $\frac{6}{12}$, $\frac{4}{12}$, $\frac{3}{12}$.

(5) Least common multiple of 3, 6, and 9=18. (Art. 119, 120.)
$\frac{18}{3}=6$, $\frac{18}{6}=3$, $\frac{18}{9}=2$;
$1\times6=6$, $1\times3=3$, $1\times2=2$. *Ans.* $\frac{6}{18}$, $\frac{3}{18}$, $\frac{2}{18}$.

(6) Least common multiple of 2, 4, and 5=20.
$\frac{20}{2}=10$, $\frac{20}{4}=5$, $\frac{20}{5}=4$;
$1\times10=10$, $3\times5=15$, $4\times4=16$. *Ans.* $\frac{10}{20}$, $\frac{15}{20}$, $\frac{16}{20}$.

(7) Least common multiple of 8, 5, and 10=40.
$\frac{40}{8}=5$, $\frac{40}{5}=8$, $\frac{40}{10}=4$;
$3\times5=15$, $4\times8=32$, $9\times4=36$. *Ans.* $\frac{15}{40}$, $\frac{32}{40}$. $\frac{36}{40}$.

(8) Least common multiple of 3, 4, and 8=24.
$\frac{24}{3}=8$, $\frac{24}{4}=6$, $\frac{24}{8}=3$;
$2\times8=16$, $3\times6=18$, $7\times3=21$. *Ans.* $\frac{16}{24}$, $\frac{18}{24}$, $\frac{21}{24}$.

(9) Least common multiple of 4, 8, and 9=72.
$\frac{72}{4}=18$, $\frac{72}{8}=9$, $\frac{72}{9}=8$;
$3\times18=54$, $5\times9=45$, $5\times8=40$. *Ans.* $\frac{54}{72}$, $\frac{45}{72}$, $\frac{40}{72}$.

(10) $\frac{9}{21}=\frac{3}{7}$. Least common multiple of 7, 14, 7, and 28=28.
$\frac{28}{7}=4$, $\frac{28}{14}=2$, $\frac{28}{7}=4$, $\frac{28}{28}=1$;
$2\times4=8$, $5\times2=10$, $3\times4=12$, $11\times1=11$.
Ans. $\frac{8}{28}$, $\frac{10}{28}$, $\frac{12}{28}$, $\frac{11}{28}$.

(11) Least common multiple of 2, 3, 4, 5, 6=60.
$\frac{60}{2}=30$, $\frac{60}{3}=20$, $\frac{60}{4}=15$, $\frac{60}{5}=12$, $\frac{60}{6}=10$:
$1\times30=30$, $2\times20=40$, $3\times15=45$, $4\times12=48$,
$5\times10=50$. *Ans.* $\frac{30}{60}$, $\frac{40}{60}$, $\frac{45}{60}$, $\frac{48}{60}$, $\frac{50}{60}$.

(12) $\frac{1}{2}$ of $\frac{7}{4}=\frac{7}{8}$. $\frac{10}{12}=\frac{5}{6}$. $2\frac{2}{5}=\frac{12}{5}$.
The fractions then are, $\frac{3}{4}$, $\frac{7}{8}$, $\frac{5}{6}$, and $\frac{12}{5}$.
Least common multiple of 4, 8, 6, and 5=120.
$\frac{120}{4}=30$, $\frac{120}{8}=15$, $\frac{120}{6}=20$, $\frac{120}{5}=24$;
$3\times30=90$, $7\times15=105$, $5\times20=100$, $12\times24=288$.
Ans. $\frac{90}{120}$, $\frac{105}{120}$, $\frac{100}{120}$, $\frac{288}{120}$.

SUGGESTION.—In working the questions, until the pupils are well acquainted with the operation, they should be required to put down the work after the manner of Ex. 1, Art. 146, Arith., as this presents the reason why the values of the fractions are not altered.

ARTICLE 147.—ADDITION.

(2) *Ans.* $\frac{6}{4}=\frac{3}{2}=1\frac{1}{2}$. **(3)** *Ans.* $\frac{10}{5}=2$. **(4)** *Ans.* $\frac{14}{7}=2$.
(5) *Ans.* $\frac{16}{8}=2$. **(6)** *Ans.* $\frac{10}{10}=1$. **(7)** *Ans.* $\frac{37}{12}=3\frac{1}{12}$.

ARTICLE 148.

(5) $4+17+29+48+76=174$; hence, sum$=\frac{174}{87}=2$ *Ans.*

(6) $11+19+29+101=160$; hence, sum$=\frac{160}{144}=\frac{10}{9}=1\frac{1}{9}$ *Ans.*

(7) $999+888+777=2664$; hence, sum$=\frac{2664}{1000}=\frac{333}{125}$
$=2\frac{83}{125}$ *Ans.*

(8) $\frac{3}{4}+\frac{5}{6}=\frac{9}{12}+\frac{10}{12}=\frac{19}{12}=1\frac{7}{12}$ *Ans.*

(9) $\frac{5}{6}+\frac{1}{9}=\frac{15}{18}+\frac{2}{18}=\frac{17}{18}$ *Ans.*

(10) $\frac{7}{8}+\frac{11}{12}=\frac{21}{24}+\frac{22}{24}=\frac{43}{24}=1\frac{19}{24}$ *Ans.*

(11) $\frac{1}{4}+\frac{7}{8}+\frac{11}{12}=\frac{6}{24}+\frac{21}{24}+\frac{22}{24}=\frac{49}{24}=2\frac{1}{24}$ *Ans.*

(12) $\frac{1}{8}+\frac{1}{9}+\frac{2}{11}=\frac{99}{792}+\frac{88}{792}+\frac{144}{792}=\frac{331}{792}$ *Ans.*

(13) $\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+\frac{1}{5}+\frac{1}{6}=\frac{30}{60}+\frac{20}{60}+\frac{15}{60}+\frac{12}{60}+\frac{10}{66}=\frac{87}{60}=\frac{29}{20}$
$=1\frac{9}{20}$ *Ans.*

(14) $\frac{1}{12}+\frac{1}{13}+\frac{1}{14}+\frac{1}{15}=\frac{455}{5460}+\frac{420}{5460}+\frac{390}{5460}+\frac{364}{5460}$
$=\frac{1629}{5460}=\frac{543}{1820}$ *Ans.*

(15) $\frac{1}{2}+\frac{1}{3}=\frac{3}{6}+\frac{2}{6}=\frac{5}{6}$; $2+3=5$; $5+\frac{5}{6}=5\frac{5}{6}$ *Ans.*

(16) $\frac{4}{5}+\frac{1}{2}+\frac{3}{4}=\frac{16}{20}+\frac{10}{20}+\frac{15}{20}=\frac{41}{20}=2\frac{1}{20}$; $7+8=15$; $15+2\frac{1}{20}=17\frac{1}{20}$ *Ans.*

(17) $\frac{2}{3}+\frac{3}{4}+\frac{3}{5}+\frac{1}{4}=\frac{40}{60}+\frac{45}{60}+\frac{36}{60}+\frac{15}{60}=\frac{136}{60}=\frac{34}{15}=2\frac{4}{15}$; $16+12+8+2=38$; $38+2\frac{4}{15}=40\frac{4}{15}$ *Ans.*

(18) $\frac{2}{3}$ of $\frac{3}{4}=\frac{6}{12}=\frac{1}{2}$; $\frac{1}{2}$ of $\frac{4}{5}$ of $\frac{5}{12}=\frac{1}{6}$; $\frac{1}{2}+\frac{1}{6}=\frac{3}{6}+\frac{1}{6}=\frac{4}{6}=\frac{2}{3}$ *Ans.*

(19) $\frac{1}{5}$ of $1\frac{1}{2}=\frac{1}{5}$ of $\frac{3}{2}=\frac{3}{10}$; $\frac{1}{9}$ of $2\frac{1}{2}=\frac{1}{9}$ of $\frac{5}{2}=\frac{5}{18}$; $\frac{11}{6}$ of $\frac{1}{2}=\frac{11}{12}$; $\frac{3}{10}+\frac{5}{18}+\frac{11}{12}+\frac{7}{8}=\frac{108}{360}+\frac{100}{360}+\frac{330}{360}+\frac{315}{360}=\frac{853}{360}=2\frac{133}{360}$ *Ans.*

(20) $\frac{2}{5}+\frac{7}{16}+\frac{7}{50}+\frac{3}{140}+\frac{3}{2800}=\frac{1120}{2800}+\frac{1225}{2800}+\frac{392}{2800}+\frac{60}{2800}+\frac{3}{2800}=\frac{2800}{2800}=1$ *Ans.*

ARTICLE 149.—SUBTRACTION.

(2) $\frac{2}{4}=\frac{1}{2}$: $\frac{2}{8}=\frac{1}{4}$: $\frac{3}{9}=\frac{1}{3}$: $\frac{5}{10}=\frac{1}{2}$: $\frac{10}{5}=2$: $\frac{8}{8}=1$.

ARTICLE 150.

(5) $\frac{8}{9}-\frac{5}{9}=\frac{3}{9}=\frac{1}{3}$ *Ans.*

(6) $\frac{15}{16}-\frac{7}{16}=\frac{8}{16}=\frac{1}{2}$ *Ans.*

(7) $\frac{3}{4}-\frac{1}{3}=\frac{9}{12}-\frac{4}{12}=\frac{5}{12}$ *Ans.*

(8) $\frac{3}{5}-\frac{1}{4}=\frac{12}{20}-\frac{5}{20}=\frac{7}{20}$ *Ans.*

(9) $\frac{1}{4}-\frac{2}{9}=\frac{9}{36}-\frac{8}{36}=\frac{1}{36}$ *Ans.*

(10) $\frac{1}{7}-\frac{1}{11}=\frac{11}{77}-\frac{7}{77}=\frac{4}{77}$ *Ans.*

(11) $\frac{5}{6}-\frac{3}{8}=\frac{20}{24}-\frac{9}{24}=\frac{11}{24}$ *Ans.*

(12) $\frac{5}{9}-\frac{1}{6}=\frac{10}{18}-\frac{3}{18}=\frac{7}{18}$ *Ans.*

(13) $\frac{4}{15}-\frac{1}{10}=\frac{8}{30}-\frac{3}{30}=\frac{5}{30}=\frac{1}{6}$ *Ans.*

(14) $\frac{8}{21}-\frac{3}{14}=\frac{16}{42}-\frac{9}{42}=\frac{7}{42}=\frac{1}{6}$ *Ans.*

(17) $4\frac{3}{4}-2\frac{1}{4}=2\frac{2}{4}=2\frac{1}{2}$ *Ans.*

(18) $8\frac{1}{3}-3\frac{2}{3}=\frac{25}{3}-\frac{11}{3}=\frac{14}{3}=4\frac{2}{3}$ *Ans.*

(19) $5\frac{2}{3}-4\frac{1}{2}=5\frac{4}{6}-4\frac{3}{6}=1\frac{1}{6}$ *Ans.*

(20) $7\frac{2}{3}-4\frac{3}{4}=\frac{23}{3}-\frac{19}{4}=\frac{92}{12}-\frac{57}{12}=\frac{35}{12}=2\frac{11}{12}$ *Ans.*

(21) $5-1\frac{1}{2}=\frac{10}{2}-\frac{3}{2}=\frac{7}{2}=3\frac{1}{2}$ *Ans.*

(22) $8-\frac{3}{7}=\frac{56}{7}-\frac{3}{7}=\frac{53}{7}=7\frac{4}{7}$ *Ans.*

(23) $11-4\frac{5}{9}=\frac{99}{9}-\frac{41}{9}=\frac{58}{9}=6\frac{4}{9}$ *Ans.*

(24) $8-3\frac{3}{13}=7\frac{13}{13}-3\frac{3}{13}=4\frac{10}{13}$ *Ans.*

(25) $\frac{3}{5}$ of $\frac{10}{1}=6$; $\frac{2}{3}$ of $\frac{6}{1}=4$; $6-4=2$ *Ans.*

(26) $\frac{1}{2}$ of $\frac{3}{4}=\frac{3}{8}$; $\frac{1}{3}$ of $\frac{5}{6}=\frac{5}{18}$; $\frac{3}{8}-\frac{5}{18}=\frac{27}{72}-\frac{20}{72}=\frac{7}{72}$ *Ans.*

(27) $\frac{2}{3}$ of $19=\frac{38}{3}$; $14\frac{1}{4}=\frac{57}{4}$; $\frac{57}{4}-\frac{38}{3}=\frac{171}{12}-\frac{152}{12}=\frac{19}{12}$
$=1\frac{7}{12}$ *Ans.*

(28) $\frac{3}{5}$ of $21=\frac{63}{5}$; $\frac{3}{8}$ of $72=27=\frac{135}{5}$; $\frac{135}{5}-\frac{63}{5}=\frac{72}{5}$
$=14\frac{2}{5}$ *Ans.*

MULTIPLICATION.

Suggestions to Teachers.—When the pupil comes to the multiplication of fractions, he will probably be surprised, at first, to find that the product is frequently less than the multiplicand, while in the multiplication of whole numbers, it is always greater. The pupil should be reminded that the multiplier shows *how many times* the multiplicand is to be taken. Thus, if the multiplier is 2, the multiplicand is to be taken two times; if 1, it is to be taken one time; if $\frac{1}{2}$, it is to be taken half a time, etc. Hence, if the multiplier be less than 1, the product will be less than the multiplicand, also, the less the value of the fraction by which we multiply, the less the product: when the multiplier becomes infinitely small, the product becomes infinitely small, and when the multiplier is 0, the product is also 0.

It should also be noticed, that to multiply by $\frac{1}{2}$, $\frac{1}{3}$, etc., is the same as to divide by 2, 3, etc.; also, that to multiply by $\frac{2}{3}$, is the same as to multiply by 2, and divide by 3, etc. Indeed, it is a general rule, *that when any result is obtained by dividing one number by another, the same result may be obtained by multiplying the given number by the divisor, written in a fractional form, and inverted.*

ARTICLE 151.—CASE I.

(14) $\frac{11}{17}\times 12=\frac{132}{17}=7\frac{13}{17}$ *Ans.* **(15)** $\frac{2}{21}\times 7=\frac{2}{\frac{21}{7}}=\frac{2}{3}$ *Ans.*

(16) $\frac{5}{36}\times9=\dfrac{5}{\frac{36}{9}}=\frac{5}{4}=1\frac{1}{4}$ *Ans.* **(17)** $\frac{25}{64}\times8=\dfrac{25}{\frac{64}{8}}=\frac{25}{8}=3\frac{1}{8}$ *Ans.*

(18) $\frac{9}{14}\times21=\dfrac{9\times21}{14}=\dfrac{9\times3}{2}=\frac{27}{2}=13\frac{1}{2}$ *Ans.*

(19) $\frac{11}{15}\times10=\dfrac{11\times10}{15}=\dfrac{11\times2}{3}=\frac{22}{3}=7\frac{1}{3}$ *Ans.*

(20) $\frac{5}{9}\times9=\dfrac{5\times9}{9}=5$ *Ans.*

(22) $18\frac{3}{4}\times8=\frac{3}{4}\times8+18\times8=6+144=150$ *Ans.*

(23) $16\frac{2}{3}\times3=\frac{2}{3}\times3+16\times3=2+48=50$ *Ans.*

(24) $10\frac{7}{9}\times7=\frac{7}{9}\times7+10\times7=\frac{49}{9}+70=75\frac{4}{9}$ *Ans.*

(25) $10\frac{5}{6}\times9=\frac{5}{6}\times9+10\times9=\frac{45}{6}+90=97\frac{1}{2}$ *Ans.*

ARTICLE 152.—CASE II.

(12) $28\times\frac{4}{7}=\dfrac{28\times4}{7}=4\times4=16$ *Ans.*

(13) $36\times\frac{7}{9}=\dfrac{36\times7}{9}=4\times7=28$ *Ans.*

(14) $50\times\frac{9}{10}=\dfrac{50\times9}{10}=5\times9=45$ *Ans.*

(15) $31\times\frac{2}{3}=\dfrac{31\times2}{3}=\frac{62}{3}=20\frac{2}{3}$ *Ans.*

(16) $29\times\frac{3}{4}=\dfrac{29\times3}{4}=\frac{87}{4}=21\frac{3}{4}$ *Ans.*

(17) $37\times\frac{4}{5}=\dfrac{37\times4}{5}=\frac{148}{5}=29\frac{3}{5}$ *Ans.*

(19) $25\times8\frac{3}{5}=\frac{25}{1}\times\frac{43}{5}=5\times43=215$ *Ans.*

(20) $45\times6\frac{1}{4}=\frac{45}{1}\times\frac{25}{4}=\frac{1125}{4}=281\frac{1}{4}$ *Ans.*

(21) $55\times9\frac{3}{7}=\frac{55}{1}\times\frac{66}{7}=\frac{3630}{7}=518\frac{4}{7}$ *Ans.*

(22) $64\times8\frac{7}{8}=\frac{64}{1}\times\frac{71}{8}=8\times71=568$ *Ans.*

ARTICLE 153.^{a}—CASE III.

(4) $\frac{9}{13}\times\frac{3}{7}=\frac{27}{91}$ *Ans.* **(5)** $\frac{11}{12}\times\frac{5}{9}=\frac{55}{108}$ *Ans.*

(6) $\frac{7}{15}\times\frac{8}{11}=\frac{56}{165}$ *Ans.* **(7)** $\frac{8}{13}\times\frac{11}{16}=\frac{11}{13\times2}=\frac{11}{26}$ *Ans.*

(9) $8\frac{3}{4}=\frac{35}{4}$. $\frac{35}{4}\times\frac{3}{7}=\frac{5\times3}{4}=\frac{15}{4}=3\frac{3}{4}$ *Ans.*

(10) $17\frac{3}{11}=\frac{190}{11}$; $\frac{9}{10}\times\frac{190}{11}=\frac{9\times19}{11}=\frac{171}{11}=15\frac{6}{11}$ *Ans.*

(11) $2\frac{1}{2}=\frac{5}{2}$; $\frac{5}{2}\times\frac{5}{2}=\frac{25}{4}=6\frac{1}{4}$ *Ans.*

(12) $16\frac{1}{2}=\frac{33}{2}$; $\frac{33}{2}\times\frac{33}{2}=\frac{1089}{4}=272\frac{1}{4}$ *Ans.*

(13) $\frac{1}{3}$ of $4=\frac{4}{3}$; $\frac{1}{6}$ of $5=\frac{5}{6}$; $\frac{4}{3}\times\frac{5}{6}=\frac{2\times5}{3\times3}=\frac{10}{9}=1\frac{1}{9}$ *Ans.*

(14) $\frac{2}{7}$ of $4\frac{1}{3}=\frac{2}{7}$ of $\frac{13}{3}=\frac{26}{21}$; $\frac{1}{3}$ of $3\frac{1}{2}=\frac{1}{3}$ of $\frac{7}{2}=\frac{7}{6}$;

$\frac{26}{21}\times\frac{7}{6}=\frac{13}{3\times3}=\frac{13}{9}=1\frac{4}{9}$ *Ans.*

(15) $\frac{3}{4}$ of $9\frac{1}{4}=\frac{3}{4}$ of $\frac{37}{4}=\frac{111}{16}$; $\frac{2}{3}$ of $17=\frac{34}{3}$;

$\frac{111}{16}\times\frac{34}{3}=\frac{37\times17}{8}=\frac{629}{8}=78\frac{5}{8}$ *Ans.*

(16) $\frac{9}{10}$ of $7\frac{1}{3}=\frac{9}{10}$ of $\frac{23}{3}=\frac{3\times11}{5}=\frac{33}{5}$; $\frac{11}{15}$ of $86\frac{3}{11}=\frac{11}{15}$ of $\frac{949}{11}$

$=\frac{949}{15}$; $\frac{33}{5}\times\frac{949}{15}=\frac{11\times949}{5\times5}=\frac{10439}{25}=417\frac{14}{25}$ *Ans.*

(17) $\frac{\cancel{3}}{1}\times\frac{5}{\cancel{2}}\times\frac{5}{7}\times\frac{\overset{5}{\cancel{10}}}{\cancel{3}}=\frac{125}{7}=17\frac{6}{7}$ *Ans.*

(18) $\frac{\cancel{7}}{\cancel{8}}\times\frac{\cancel{3}}{\cancel{10}}\times\frac{\cancel{8}}{9}\times\frac{\cancel{5}}{\cancel{6}}\times\frac{\cancel{2}}{\cancel{3}}\times\frac{\cancel{6}}{\cancel{7}}=\frac{1}{9}$ *Ans.*

(19) $\frac{1}{\cancel{4}}\times\frac{\cancel{9}}{\cancel{7}}\times\frac{\cancel{4}}{\cancel{5}}\times\frac{\cancel{7}}{\cancel{9}}\times\frac{\cancel{5}}{4}\times\frac{2}{3}\times\frac{6}{1}=\frac{12}{12}=1$ *Ans.*

(20) $\frac{\cancel{6}}{\cancel{7}}\times\frac{\cancel{4}}{9}\times\frac{\cancel{7}}{\cancel{4}}\times\frac{1}{\cancel{6}}\times\frac{\cancel{3}}{\cancel{4}}\times\frac{5}{\cancel{6}}\times\frac{\cancel{2}}{\cancel{5}}\times\frac{\cancel{20}}{1}=\frac{5}{9}$ *Ans.*

(21) $\frac{\cancel{5}}{\cancel{2}}\times\frac{\overset{8}{\cancel{32}}}{\cancel{5}}\times\frac{\cancel{13}}{\cancel{4}}\times\frac{\cancel{7}}{\cancel{13}}\times\frac{\cancel{2}}{1}\times\frac{3}{\cancel{7}}=24$ *Ans.*

ARTICLE 153.^{b}

(1) $1\frac{1}{5}=\frac{6}{5}$; $2\frac{1}{3}=\frac{7}{3}$; $\frac{\overset{2}{\cancel{6}}}{5}\times\frac{7}{\cancel{3}}=\frac{14}{5}=2\frac{4}{5}$ cts. *Ans.*

(2) $\overset{\$}{\frac{2}{3}}\times3=\overset{\$}{2}$; $\overset{\$}{\frac{2}{3}}\times5=\frac{10}{3}=\overset{\$}{3\frac{1}{3}}$; $\overset{\$}{\frac{2}{3}}\times7=\frac{14}{3}=\overset{\$}{4\frac{2}{3}}$;
$\frac{\cancel{2}}{3}\times\frac{13}{\cancel{2}}=\frac{13}{3}=\overset{\$}{4\frac{1}{3}}$; $\frac{2}{3}\times\frac{23}{4}=\frac{23}{6}=\overset{\$}{3\frac{5}{6}}$ *Ans.*

(3) $4\frac{4}{5}=\frac{24}{5}$; $3\frac{1}{3}=\frac{10}{3}$; $\frac{\overset{8}{\cancel{24}}}{\cancel{5}}\times\frac{\overset{2}{\cancel{10}}}{\cancel{3}}=16$ cts. *Ans.*

(4) $18\frac{3}{4}=\frac{75}{4}$; $3\frac{1}{5}=\frac{16}{5}$; $\frac{\overset{15}{\cancel{75}}}{\cancel{4}}\times\frac{\overset{4}{\cancel{16}}}{5}=\$60$ *Ans.*

(5) $1\frac{2}{3}=\frac{5}{3}$; $\frac{\cancel{3}}{\underset{4}{\cancel{20}}}\times\frac{\cancel{5}}{\cancel{3}}=\$\frac{1}{4}$ *Ans.*

(6) $2\frac{1}{2}=\frac{5}{2}$; $\frac{4}{\cancel{5}}\times\frac{\cancel{5}}{2}=\frac{4}{2}=\$2$ *Ans.*

(7) $5\frac{5}{9}=\frac{50}{9}$; $1\frac{1}{5}=\frac{6}{5}$; $\frac{\overset{10}{\cancel{50}}}{\underset{3}{\cancel{9}}}\times\frac{\overset{2}{\cancel{6}}}{\cancel{5}}=\frac{20}{3}=\$6\frac{2}{3}$ *Ans.*

(8) $5\frac{1}{2}=\frac{11}{2}$; $7\frac{3}{4}=\frac{31}{4}$; $\frac{11}{2}\times\frac{31}{4}=\frac{341}{8}=42\frac{5}{8}$ mi. *Ans.*

(9) $\frac{3}{5}$ of $\frac{2}{3}=\frac{\cancel{3}}{5}\times\frac{2}{\cancel{3}}=\frac{2}{5}$ *Ans.*

(10) $6\frac{3}{4}=\frac{27}{4}$; $5\frac{1}{2}=\frac{11}{2}$; $\frac{2}{9}$ of $\frac{11}{2}=\frac{11}{9}$; $\frac{\overset{3}{\cancel{27}}}{4}\times\frac{11}{\cancel{9}}=\frac{33}{4}=\$8\frac{1}{4}$ [*Ans.*

(11) $\frac{\cancel{3}}{\cancel{7}}\times\frac{5}{\cancel{9}}\times\frac{\overset{11}{\cancel{33}}}{\cancel{2}}\times\frac{\cancel{2}}{\cancel{3}}\times\frac{\cancel{7}}{8}\times\frac{\overset{5}{\cancel{15}}}{1}=\frac{5\times11\times5}{8}=\frac{275}{8}=34\frac{3}{8}$ *Ans.*

ARTICLE 154.—CASE I.

REMARK.—In dividing a fraction by a whole number, the quotient will always be less than the dividend, as in the division of whole numbers; thus, to divide $\frac{1}{3}$ by 4, we really find what *part of a time* 4 is contained in $\frac{1}{3}$; we also find the $\frac{1}{4}$ of $\frac{1}{3}$, since to divide by 4, is the same thing as to multiply by $\frac{1}{4}$.

In the solution of the following questions, we have not marked the canceled factors, by drawing lines across them, on account of the extra space occupied by this kind of type: in operations performed on the slate, or at the blackboard, the canceled factors should always be marked, in order to avoid mistakes.

(5) $\frac{12}{16}\div4=\frac{3}{16}$ *Ans.* (6) $\frac{9}{11}\div3=\frac{3}{11}$ *Ans.*

(7) $\frac{3}{20}\div5=\frac{3}{100}$ *Ans.* (8) $\frac{14}{25}\div7=\frac{2}{25}$ *Ans.*

(9) $\frac{16}{25}\div4=\frac{4}{25}$ *Ans.* (10) $\frac{1}{2}\div5=\frac{1}{10}$ *Ans.*

(12) $3\frac{3}{4}=\frac{15}{4}$; $\frac{15}{4}\div5=\frac{3}{4}$ *Ans.* (13) $8\frac{2}{5}=\frac{42}{5}$; $\frac{42}{5}\div7=\frac{6}{5}=1\frac{1}{5}$ *Ans.*

(14) $\frac{18}{19}\div9=\frac{2}{19}$ *Ans.* (15) $4\frac{4}{5}=\frac{24}{5}$; $\frac{24}{5}\div8=\frac{3}{5}$ *Ans.*

(16) $\frac{3}{4}\div5=\frac{3}{20}$ *Ans.* (17) $12\frac{4}{7}=\frac{88}{7}$; $\frac{88}{7}\div11=\frac{8}{7}=1\frac{1}{7}$ *Ans.*

(18) $3\frac{2}{3}=\frac{11}{3}$; $\frac{11}{3}\div7=\frac{11}{21}$ *Ans.*

(19) $47\frac{2}{5}=\frac{237}{5}$; $\frac{237}{5}\div15=\frac{237}{5}\times\frac{1}{15}=\frac{79}{5\times5}=\frac{79}{25}=3\frac{4}{25}$ *Ans.*

(20) $130\frac{2}{3}=\frac{392}{3}$; $\frac{392}{3}\div18=\frac{392}{3}\times\frac{1}{18}=\frac{196}{3\times9}=\frac{196}{27}=7\frac{7}{27}$ *Ans.*

ARTICLE 155.—CASE II.

(4) $4\div\frac{2}{5}=\frac{4\times5}{2}=10$ *Ans.* **(5)** $16\div\frac{3}{4}=\frac{16\times4}{3}=\frac{64}{3}=21\frac{1}{3}$ *Ans.*

(6) $8\div\frac{2}{15}=\frac{8\times15}{2}=4\times15=60$ *Ans.*

(7) $6\div\frac{3}{7}=\frac{6\times7}{3}=2\times7=14$ *Ans.*

(8) $13\div\frac{3}{5}=\frac{13\times5}{3}=\frac{65}{3}=21\frac{2}{3}$ *Ans.*

(9) $21\div\frac{7}{11}=\frac{21\times11}{7}=3\times11=33$ *Ans.*

(11) $2\frac{2}{5}=\frac{12}{5}$; $6\div\frac{12}{5}=\frac{6\times5}{12}=\frac{5}{2}=2\frac{1}{2}$ yd. *Ans.*

(12) $3\frac{3}{4}=\frac{15}{4}$; $30\div\frac{15}{4}=\frac{30\times4}{15}=2\times4=8$ lb. *Ans.*

(13) $4\frac{3}{7}=\frac{31}{7}$; $50\div\frac{31}{7}=\frac{50\times7}{31}=\frac{350}{31}=11\frac{9}{31}$ *Ans.*

(14) $5\frac{4}{9}=\frac{49}{9}$; $56\div\frac{49}{9}=\frac{56\times9}{49}=\frac{8\times9}{7}=\frac{72}{7}=10\frac{2}{7}$ *Ans.*

ARTICLE 156.—CASE III.

REMARK.—Pupils are often at a loss to understand, why it is that the quotient of one proper fraction, divided by another, is sometimes a whole number, or greater than unity. The teacher should be careful to explain this subject, by means of familiar examples, such as may be found in Ray's Arithmetic, 2d Book, Section xxi. It should also be shown, that if we take any dividend, and divide it by different numbers, that as the divisor becomes less, the quotient becomes greater; so that, by making the divisor sufficiently small, the quotient may be made as large as we please. Thus, the quotient of $\frac{1}{2}$ divided by $\frac{1}{4}$, is 2; by $\frac{1}{8}$, is 4; by $\frac{1}{16}$, is 8; by $\frac{1}{4000000}$, is 2000000, etc. It is on this principle, that mathematicians, say, that the quotient of any number, divided by 0, is infinitely large.

(5) $\frac{3}{4}\div\frac{1}{4}=\frac{3}{4}\times\frac{4}{1}=3$ *Ans.* (6) $\frac{1}{2}\div\frac{1}{4}=\frac{1}{2}\times\frac{4}{1}=2$ *Ans*

(7) $\frac{1}{4}\div\frac{1}{2}=\frac{1}{4}\times\frac{2}{1}=\frac{1}{2}$ *Ans.* (8) $\frac{5}{6}\div\frac{2}{3}=\frac{5}{6}\times\frac{3}{2}=\frac{5}{4}=1\frac{1}{4}$ *Ans.*

(9) $\frac{1}{2}\div\frac{1}{3}=\frac{1}{2}\times\frac{3}{1}=\frac{3}{2}=1\frac{1}{2}$ *Ans.*

(10) $2\frac{1}{2}=\frac{5}{2}$; $\frac{5}{2}\div\frac{1}{16}=\frac{5}{2}\times\frac{16}{1}=40$ *Ans.*

(11) $4\frac{1}{2}=\frac{9}{2}$; $1\frac{1}{3}=\frac{4}{3}$; $\frac{9}{2}\div\frac{4}{3}=\frac{9}{2}\times\frac{3}{4}=\frac{27}{8}=3\frac{3}{8}$ *Ans.*

(12) $4\frac{3}{4}=\frac{19}{4}$; $5\frac{1}{8}=\frac{41}{8}$; $\frac{19}{4}\div\frac{41}{8}=\frac{19}{4}\times\frac{8}{41}=\dfrac{19\times2}{41}=\frac{38}{41}$ *Ans.*

(13) $2\frac{1}{4}=\frac{9}{4}$; $7\frac{1}{2}=\frac{15}{2}$; $\frac{9}{4}\div\frac{15}{2}=\frac{9}{4}\times\frac{2}{15}=\frac{3}{2}\times\frac{1}{5}=\frac{3}{10}$ *Ans.*

(14) $\frac{1}{2}\div\frac{1}{50}=\frac{1}{2}\times\frac{50}{1}=25$ *Ans.*

(15) $\frac{3}{5}$ of $\frac{8}{9}=\frac{8}{15}$; $\frac{6}{7}$ of $\frac{3}{4}=\frac{9}{14}$; $\frac{8}{15}\div\frac{9}{14}=\frac{8}{15}\times\frac{14}{9}=\frac{112}{135}$ *Ans.*

(16) $\frac{1}{3}$ of $\frac{41}{8}=\frac{41}{24}$; $\frac{3}{4}$ of $\frac{35}{2}=\frac{105}{8}$; $\frac{41}{24}\times\frac{8}{105}=\dfrac{41}{3\times105}=\frac{41}{315}$ [*Ans.*

ARTICLE 157.

(2) $\frac{14}{15}\div21=\frac{14}{15}\times\frac{1}{21}=\frac{2}{15}\times\frac{1}{3}=\frac{2}{45}$ *Ans.*

(3) $8\frac{5}{9}=\frac{77}{9}$; $\frac{77}{9}\div35=\frac{77}{9}\times\frac{1}{35}=\frac{11}{9}\times\frac{1}{5}=\frac{11}{45}$ *Ans.*

(4) $8\div\frac{16}{17}=\frac{8}{1}\times\frac{17}{16}=\frac{17}{2}=8\frac{1}{2}$ *Ans.*

(5) $8\frac{3}{4}=\frac{35}{4}$; $14\div\frac{35}{4}=\frac{14}{1}\times\frac{4}{35}=\frac{2}{1}\times\frac{4}{5}=\frac{8}{5}=1\frac{3}{5}$ *Ans.*

(6) $\frac{21}{32}\div\frac{35}{48}=\frac{21}{32}\times\frac{48}{35}=\frac{3}{2}\times\frac{3}{5}=\frac{9}{10}$ *Ans.*

(7) $12\frac{5}{6}=\frac{77}{6}$; $4\frac{1}{8}=\frac{33}{8}$; $\frac{77}{6}\times\frac{8}{33}=\frac{7}{3}\times\frac{4}{3}=\frac{28}{9}=3\frac{1}{9}$ *Ans.*

(8)
$$\begin{array}{r|l} 6\ \cancel{18} & 5 \\ \cancel{5} & \cancel{2} \\ \cancel{10} & \cancel{123}\ \ \cancel{3} \\ \cancel{1} & \cancel{5} \\ \cancel{41} & \cancel{5} \\ \hline 6 & 5=\frac{5}{6}\ \textit{Ans.} \end{array}$$

In solving this example we find $12\frac{3}{10}=\frac{123}{10}$, and $8\frac{1}{5}=\frac{41}{5}$. When the latter is inverted, 41 becomes a divisor, and, therefore, is placed on the left.

ARTICLE 158.^{a}

(2) $\dfrac{\frac{6}{7}}{\frac{11}{5}}=\frac{6}{7}\times\frac{5}{11}=\frac{30}{77}$ *Ans.* (3) $\dfrac{\frac{2}{3}}{5}=\frac{2}{3}\times\frac{1}{5}=\frac{2}{15}$ *Ans.*

(4) $\dfrac{2}{3\frac{2}{3}}=\dfrac{2}{\frac{11}{3}}=\frac{2}{1}\times\frac{3}{11}=\frac{6}{11}$ *Ans.*

(5) $\dfrac{3\frac{1}{8}}{4\frac{5}{7}}=\frac{25}{8}\div\frac{33}{7}=\frac{25}{8}\times\frac{7}{33}=\frac{175}{264}$ *Ans.*

(6) $\dfrac{2\frac{1}{3}}{4\frac{1}{2}}=\frac{7}{3}\div\frac{9}{2}=\frac{7}{3}\times\frac{2}{9}=\frac{14}{27}$ *Ans.*

(7) $\dfrac{3\frac{3}{4}}{5\frac{5}{8}}=\frac{15}{4}\div\frac{45}{8}=\frac{15}{4}\times\frac{8}{45}=\frac{1}{1}\times\frac{2}{3}=\frac{2}{3}$ *Ans.*

(8) $\dfrac{1\frac{1}{2}}{2\frac{1}{3}}=\frac{3}{2}\times\frac{3}{7}=\frac{9}{14}$; $\dfrac{\frac{3}{7}}{4\frac{1}{2}}=\frac{3}{7}\times\frac{2}{9}=\frac{2}{21}$; $\frac{9}{14}\times\frac{2}{21}=\dfrac{3}{7\times7}=\frac{3}{49}$ *Ans.*

(9) $7\frac{8}{15}=\frac{113}{15}$; $9\frac{5}{12}=\frac{113}{12}$;
$2\frac{1}{9}=\frac{19}{9}$; $3\frac{2}{15}=\frac{47}{15}$; $1\frac{1}{4}=\frac{5}{4}$.

$$\begin{array}{rr|ll}
 & \cancel{15} & \cancel{113} & \\
 & \cancel{113} & \cancel{12} & \cancel{4} \\
\cancel{3} & \cancel{9} & 19 & \\
 & 47 & \cancel{15} & \\
 & \cancel{4} & \cancel{5} & \\
 & \cancel{5} & \cancel{6} & 2 \\
\hline
 & 47 & \multicolumn{2}{l}{19\times2=\frac{38}{47}\ \textit{Ans.}}
\end{array}$$

(10) $1\frac{2}{3}=\frac{5}{3}$; $2\frac{1}{2}=\frac{5}{2}$;
$5\frac{1}{7}=\frac{36}{7}$; $84\frac{6}{7}=\frac{594}{7}$

$$\begin{array}{rr|ll}
 & 3 & \cancel{5} & \\
 & \cancel{5} & \cancel{2} & \\
\cancel{18} & \cancel{36} & \cancel{7} & \\
 & \cancel{7} & \cancel{594} & 33 \\
\hline
 & 3 & \multicolumn{2}{l}{33=11\ \textit{Ans.}}
\end{array}$$

ARTICLE 158.^{b}

(1) $3\frac{1}{4}\div\frac{1}{2}=\frac{13}{4}\times\frac{2}{1}=\frac{13}{2}=6\frac{1}{2}$ yd. *Ans.*

(2) $2\frac{3}{10}\div\frac{3}{5}=\frac{23}{10}\times\frac{5}{3}=\frac{23}{6}=3\frac{5}{6}$ lb. *Ans.*

(3) $\frac{1}{2}\div2=\frac{1}{4}$: $\frac{1}{2}\div\frac{1}{2}=\frac{1}{2}\times\frac{2}{1}=1$: $\frac{1}{2}\div\frac{1}{4}=\frac{1}{2}\times\frac{4}{1}=2$: $\frac{1}{2}\div\frac{1}{6}=\frac{1}{2}\times\frac{6}{1}=3$: $\frac{1}{2}\div\frac{1}{8}=\frac{1}{2}\times\frac{8}{1}=4$: $\frac{1}{2}\div\frac{1}{1000}=\frac{1}{2}\times\frac{1000}{1}=500$ *Ans.*

(4) $42\frac{1}{2}\div3\frac{3}{4}=\frac{85}{2}\div\frac{15}{4}=\frac{85}{2}\times\frac{4}{15}=\frac{17}{1}\times\frac{2}{3}=\frac{34}{3}=11\frac{1}{3}$ yd. *Ans.*

(5) $10\div\frac{3}{8}=\frac{10}{1}\times\frac{8}{3}=\frac{80}{3}=26\frac{2}{3}$ *Ans.*

(6) $\frac{3}{7}$ of $1\frac{1}{2}=\frac{3}{7}$ of $\frac{3}{2}=\frac{9}{14}$; $3\frac{3}{7}=\frac{24}{7}$;

$\frac{24}{7}\div\frac{9}{14}=\frac{24}{7}\times\frac{14}{9}=\frac{8}{1}\times\frac{2}{3}=\frac{16}{3}=5\frac{1}{3}$ *Ans.*

(7) $\frac{4}{11}$ of $27\frac{1}{2}=\frac{4}{11}$ of $\frac{55}{2}=10$; $\frac{3}{10}$ of $21\frac{1}{4}=\frac{3}{10}$ of $\frac{85}{4}$

$=\frac{3}{2}\times\frac{17}{4}=\frac{51}{8}$; $10\div\frac{51}{8}=\frac{10}{1}\times\frac{8}{51}=\frac{80}{51}=1\frac{29}{51}$ *Ans.*

(8) $\dfrac{1\frac{1}{2}}{\frac{2}{3}}=\frac{3}{2}\times\frac{3}{2}=\frac{9}{4}$; $\dfrac{2\frac{2}{5}}{2\frac{1}{6}}=\frac{12}{5}\times\frac{6}{13}=\frac{72}{65}$;

$\frac{9}{4}\div\frac{72}{65}=\frac{9}{4}\times\frac{65}{72}=\frac{1}{4}\times\frac{65}{8}=\frac{65}{32}=2\frac{1}{32}$ *Ans.*

ARTICLE 159.

(1)

$$\begin{array}{rr} & \$\ \text{cts.} \\ & 16.06\frac{1}{4} \\ & 9.12\frac{1}{2} \\ & 5.43\frac{3}{4} \\ & 2.81\frac{1}{4} \\ \cline{2-2} \textit{Ans.} & \$33.43\frac{3}{4} \end{array}$$

(2)

$$\begin{array}{rr} & \$\ \text{cts.} \\ & 9.12\frac{1}{2} \\ & 4.43\frac{3}{4} \\ & 0.37\frac{1}{2} \\ & 1.62\frac{1}{2} \\ \cline{2-2} \textit{Ans.} & \$15.56\frac{1}{4} \end{array}$$

(3)

$$\begin{array}{rr} & \$\ \text{cts.} \\ & 50.25 \\ & 27.18\frac{3}{4} \\ \cline{2-2} \textit{Ans.} & \$23.06\frac{1}{4} \end{array}$$

(4)

$$\begin{array}{rr} & \$\ \text{cts.} \\ & 32.31\frac{1}{4} \\ & 15.12\frac{1}{2} \\ \cline{2-2} \textit{Ans.} & \$17.18\frac{3}{4} \end{array}$$

(5)

$$\begin{array}{rr} & \$\ \text{cts.} \\ & 5.81\frac{1}{4} \\ & 1.18\frac{3}{4} \\ \cline{2-2} \textit{Ans.} & \$4.62\frac{1}{2} \end{array}$$

(6) $12\frac{1}{2}\times9=108+4\frac{1}{2}=112\frac{1}{2}$ cts.$=\$1.12\frac{1}{2}$ *Ans.*

(7) $6\frac{1}{4}\times21=126+5\frac{1}{4}=131\frac{1}{4}$ cts.$=\$1.31\frac{1}{4}$ *Ans.*

(8) $\$3.18\frac{3}{4}\times15=47.70+.11\frac{1}{4}=\$47.81\frac{1}{4}$ *Ans.*

(9) $62\frac{1}{2}\times5\frac{1}{2}=\frac{125}{2}\times\frac{11}{2}=\frac{1375}{4}=343\frac{3}{4}$ cts.$=\$3.43\frac{3}{4}$ *Ans.*

(10) $18\frac{3}{4}=\frac{75}{4}$: $12\frac{1}{2}=\frac{25}{2}$: $\frac{75}{4}\times\frac{25}{2}=\frac{1875}{8}=234\frac{3}{8}=\$2.34\frac{3}{8}$ *Ans.*

(11) $16\frac{2}{3}=\frac{50}{3}$: $13\frac{1}{2}=\frac{27}{2}$: $\frac{50}{3}\times\frac{27}{2}=25\times9=225$ cts$=\$2.25$ *Ans.*

(12) $\$3.37\frac{1}{2}\times10\frac{1}{4}=\frac{675}{2}\times\frac{41}{4}=\frac{27675}{8}=3459\frac{3}{8}$ cts.$=\$34.59\frac{3}{8}$ [*Ans.*

(13) $17\frac{2}{3}=\frac{53}{3}$: $375\times\frac{53}{3}=125\times53=6625$ cts.$=\$66.25$ *Ans.*

(14) $225\div 18\frac{3}{4}=\frac{225}{1}\times\frac{4}{75}=\frac{3}{1}\times\frac{4}{1}=12$ yd. *Ans.*

(15) $581\frac{1}{4}\div 37\frac{1}{2}=\frac{2325}{4}\times\frac{2}{75}=\frac{31}{2}\times\frac{1}{1}=15\frac{1}{2}$ bu. *Ans.*

(16) $\$11.56\frac{1}{4}\div 5=\$2.31\frac{1}{4}$ *Ans.*

(17) $\$31.06\frac{1}{4}\div 7=\$4.43\frac{3}{4}$ *Ans.*

(18) $\overset{\text{mi.}}{5}\times 8=\overset{\text{fur.}}{40} : \overset{\text{fur.}}{40}\times 40=\overset{\text{rd.}}{1600} : \overset{\text{rd.}}{1600}\times 16\frac{1}{2}=\overset{\text{ft.}}{26400} :$
$\overset{\text{ft.}}{26400}\times 12=\overset{\text{in.}}{316800}$ *Ans.*

(19) $\overset{\text{mi.}}{2}\times 8=\overset{\text{fur.}}{16} : \overset{\text{fur.}}{16}\times 40+\overset{\text{rd.}}{2}=\overset{\text{rd.}}{642} : \overset{\text{rd.}}{642}\times 16\frac{1}{2}+\overset{\text{ft.}}{2}=\overset{\text{ft.}}{10595}$ *Ans.*

(21) $\overset{\text{ft.}}{15875}\div 16\frac{1}{2}=\overset{\text{rd.}}{962}\ \overset{\text{ft.}}{2} : \overset{\text{rd.}}{962}\div 40=\overset{\text{fur.}}{24}\ \overset{\text{rd.}}{2} : \overset{\text{fur.}}{24}\div 8=\overset{\text{mi.}}{3}.$
Ans. 3 mi. 2 rd. 2 ft.

(22) $\overset{\text{in.}}{142634}\div 12=\overset{\text{ft.}}{11886}\ \overset{\text{in.}}{2} : \overset{\text{ft.}}{11886}\div 3=\overset{\text{yd.}}{3962} :$
$\overset{\text{yd.}}{3962}\div 5\frac{1}{2}=\overset{\text{rd.}}{720}\ \overset{\text{yd.}}{2} : \overset{\text{rd.}}{720}\div 40=\overset{\text{fur.}}{18} : \overset{\text{fur.}}{18}\div 8=\overset{\text{mi.}}{2}\ \overset{\text{fur.}}{2}.$
Ans. 2 mi. 2 fur. 2 yd. 2 in.

(23) $\overset{\text{mi.}}{2}=\overset{\text{in.}}{126720} : \overset{\text{ft.}}{2}\ \overset{\text{in.}}{8}=\overset{\text{in.}}{32} : \overset{\text{in.}}{126720}\div 32=\overset{\text{in.}}{3960}$ *Ans.*

(24) $\overset{\text{mi.}}{65}=\overset{\text{in.}}{4118400} : \overset{\text{ft.}}{9}\ \overset{\text{in.}}{2}=\overset{\text{in.}}{110} : \overset{\text{in.}}{4118400}\div 110=\overset{\text{in.}}{37440}$ *Ans.*

(25) $\overset{\text{A.}}{1}\times 4+\overset{\text{R.}}{3}=\overset{\text{R.}}{7} : \overset{\text{R.}}{7}\times 40+\overset{\text{P.}}{16}=\overset{\text{P.}}{296} : \overset{\text{P.}}{296}\times 30\frac{1}{4}+\overset{\text{sq.yd.}}{25}=\overset{\text{sq.yd.}}{8979}$ *Ans.*

(26) $\overset{\text{sq.yd.}}{7506}\div 30\frac{1}{4}=\overset{\text{P.}}{248}\ \overset{\text{sq.yd.}}{4} : \overset{\text{P.}}{248}\div 40=\overset{\text{R.}}{6}\ \overset{\text{P.}}{8} :$
$\overset{\text{R.}}{6}\div 4=\overset{\text{A.}}{1}\ \overset{\text{R.}}{2}.$ *Ans.* $\overset{\text{A.}}{1}\ \overset{\text{R.}}{2}\ \overset{\text{P.}}{8}\ \overset{\text{sq.yd.}}{4}$

(27) $\overset{\text{ch.}}{5}\ \overset{\text{l.}}{15}=\overset{\text{l.}}{515}\quad \overset{\text{in.}}{7\frac{92}{100}}=\overset{\text{in.}}{\frac{792}{100}} : \overset{\text{in.}}{\frac{792}{100}}\times\overset{\text{in.}}{515}=\frac{407880}{100}=\overset{\text{in.}}{4078\frac{4}{5}}$ *Ans.*

(28) $40\frac{1}{2}=\frac{81}{2}$: $\frac{81}{2}\times32=81\times16=\overset{\text{P.}}{1296}$: $\overset{\text{P.}}{1296}\div40$
$=\overset{\text{R.}}{32}\ \ \overset{\text{P.}}{16}$: $\overset{\text{R.}}{32}\div4=\overset{\text{A.}}{8}$. *Ans.* $\overset{\text{A.}}{8}\ \ \overset{\text{P.}}{16}$.

(29) $\overset{\text{da.}}{365\frac{1}{4}}\times4=\overset{\text{da.}}{1461}$: $\overset{\text{da.}}{1461}\times24=\overset{\text{hr.}}{35064}$ *Ans.*

(30) $\overset{\text{hr.}}{914092}\div24=\overset{\text{da.}}{38087}\ \ \overset{\text{hr.}}{4}$: $\overset{\text{da.}}{38087}\div365\frac{1}{4}=\overset{\text{yr.}}{104}\ \ \overset{\text{da.}}{101}$:
$\overset{\text{yr.}}{104}\div100=\overset{\text{cen.}}{1}\ \ \overset{\text{yr.}}{4}$. *Ans.* $\overset{\text{cen.}}{1}\ \ \overset{\text{yr.}}{4}\ \ \overset{\text{da.}}{101}\ \ \overset{\text{hr.}}{4}$.

(31) $238545\div31=\overset{\text{da.}}{7695}$: $7695\div365\frac{1}{4}=30780\div1461=21$ yr., and 99 quarter days remaining, which reduced to days, by dividing by 4, makes $24\frac{3}{4}$ days. *Ans.* 21 yr. $24\frac{3}{4}$ da.

Article 160.—Case I.

(3) $\overset{\text{lb.}}{\frac{1}{28}}\times16=\frac{16}{28}=\overset{\text{oz.}}{\frac{4}{7}}$ *Ans.*

(4) $\overset{\text{lb.}}{\frac{1}{16}}\times12=\frac{12}{16}=\overset{\text{oz.}}{\frac{3}{4}}$ *Ans.*

(5) $\overset{\text{yd.}}{\frac{1}{20}}\times4=\overset{\text{qr.}}{\frac{1}{5}}$: $\overset{\text{qr.}}{\frac{1}{5}}\times4=\overset{\text{na.}}{\frac{4}{5}}$ *Ans.*

(6) $\overset{\text{A.}}{\frac{7}{1280}}\times4=\overset{\text{R.}}{\frac{7}{320}}$: $\overset{\text{R.}}{\frac{7}{320}}\times40=\overset{\text{P.}}{\frac{7}{8}}$ *Ans.*

(7) $\overset{\$}{\frac{3}{350}}\times100=\frac{300}{350}=\overset{\text{ct.}}{\frac{6}{7}}$ *Ans.*

(8) $\overset{\text{da.}}{\frac{1}{1584}}\times24=\overset{\text{hr.}}{\frac{1}{66}}$: $\overset{\text{hr.}}{\frac{1}{66}}\times60=\frac{60}{66}=\overset{\text{min.}}{\frac{10}{11}}$ *Ans.*

(9) $\overset{\text{bu.}}{\frac{3}{320}}\times4=\overset{\text{pk.}}{\frac{3}{80}}$: $\overset{\text{pk.}}{\frac{3}{80}}\times8=\overset{\text{qt.}}{\frac{3}{10}}$: $\overset{\text{qt.}}{\frac{3}{10}}\times2=\overset{\text{pt.}}{\frac{3}{5}}$ *Ans.*

Article 161.—Case II.

(3) $\overset{\text{na.}}{\frac{4}{5}}\div4=\overset{\text{qr.}}{\frac{1}{5}}$: $\overset{\text{qr.}}{\frac{1}{5}}\div4=\overset{\text{yd.}}{\frac{1}{20}}$ *Ans.*

(4) $\overset{\text{gr.}}{\frac{3}{5}}\div24=\overset{\text{pwt.}}{\frac{1}{40}}$: $\overset{\text{pwt.}}{\frac{1}{40}}\div20=\overset{\text{oz.}}{\frac{1}{800}}$: $\overset{\text{oz.}}{\frac{1}{800}}\div12=\overset{\text{lb.}}{\frac{1}{9600}}$ *Ans.*

℈ ʒ ʒ ℥ ℥ ℔
(5) $\frac{4}{9} \div 3 = \frac{4}{27}$: $\frac{4}{27} \div 8 = \frac{1}{54}$: $\frac{1}{54} \div 12 = \frac{1}{648}$ *Ans.*

pt. qt. qt. pk. pk. bu.
(6) $\frac{3}{5} \div 2 = \frac{3}{10}$: $\frac{3}{10} \div 8 = \frac{3}{80}$: $\frac{3}{80} \div 4 = \frac{3}{320}$ *Ans.*

oz. lb. lb. qr. qr. cwt.
(7) $\frac{4}{7} \div 16 = \frac{1}{28}$: $\frac{1}{28} \div 25 = \frac{1}{700}$: $\frac{1}{700} \div 4 = \frac{1}{2800}$ *Ans.*

in. in. na. na. qr. qr. E.En.
(8) $\frac{3}{5} \div 2\frac{1}{4} = \frac{3}{5} \times \frac{4}{9} = \frac{4}{15}$: $\frac{4}{15} \div 4 = \frac{1}{15}$: $\frac{1}{15} \div 5 = \frac{1}{75}$ *Ans.*

min. hr. hr. da.
(9) $\frac{8}{9} \div 60 = \frac{2}{135}$: $\frac{2}{135} \div 24 = \frac{1}{1620}$ *Ans.*

dr. oz. oz. lb. lb. qr.
(10) $\frac{5}{7} \div 16 = \frac{5}{112}$: $\frac{5}{112} \div 16 = \frac{5}{1792}$ $\frac{5}{1792} \div 25 = \frac{1}{8960}$ *Ans.*

ARTICLE 162.—CASE III.

cts. cts.
(3) $\frac{3}{5}$ of $100 = \frac{300}{5} = 60$ *Ans.*

fur. fur. fur. rd. rd. fur. rd.
(4) $\frac{2}{5}$ of $8 = \frac{16}{5} = 3\frac{1}{5}$: $\frac{1}{5} = \frac{1}{5}$ of $40 = 8$. *Ans.* 3 8.

oz. oz. oz. pwt. pwt. oz. pwt.
(5) $\frac{4}{5}$ of $12 = \frac{48}{5} = 9\frac{3}{5}$: $\frac{3}{5} = \frac{3}{5}$ of $20 = \frac{60}{5} = 12$. *Ans.* 9 12.

oz. oz. oz. oz. dr. dr. oz. dr.
(6) $\frac{4}{7}$ of $16 = \frac{64}{7} = 9\frac{1}{7}$: $\frac{1}{7} = \frac{1}{7}$ of $16 = \frac{16}{7} = 2\frac{2}{7}$. *Ans.* 9 $2\frac{2}{7}$.

R. R. R. P. P. R. P.
(7) $\frac{5}{8}$ of $4 = \frac{5}{2} = 2\frac{1}{2}$: $\frac{1}{2} = \frac{1}{2}$ of $40 = 20$. *Ans.* 2 20.

hhd. hhd. hhd. gal. gal. gal. qt. qt.
(8) $\frac{7}{8}$ of $4 = \frac{7}{2} = 3\frac{1}{2}$: $\frac{1}{2} = \frac{1}{2}$ of $63 = 31\frac{1}{2}$: $\frac{1}{2} = \frac{1}{2}$ of $4 = 2$.
Ans. 3 hhd. 31 gal. 2 qt.

ARTICLE 163.—CASE IV.

hr. min. min. da. min.
(5) 13 30=810 : 1=1440 : $\frac{810}{1440} = \frac{9}{16}$ *Ans.*

fur. rd. rd. m. rd.
(6) 3 25=145 : 1=320 : $\frac{145}{320} = \frac{29}{64}$ *Ans.*

ft. in. in. yd. in.
(7) 2 8=32 : 1=36 : $\frac{32}{36}=\frac{8}{9}$ *Ans.*

(8) $\frac{96}{432}=\frac{2}{9}$ *Ans.*, that is, 96 pages are $\frac{2}{9}$ of 432 pages.

mi. fur. rd. rd. mi. fur. rd. rd.
(9) 15 3 3=4923 : 35 7 7=11487 : $\frac{4923}{11487}=\frac{3}{7}$ *Ans.*

A. P. P. A. R. P. P.
(10) 168 28=26908 : 37 2 14=6014 : $\frac{6014}{26908}=\frac{97}{434}$ *Ans.*

oz. dr. dr. lb. dr.
(11) 7 $1\frac{7}{9}=\frac{1024}{9}$: 1=$\frac{2304}{9}$: $\frac{1024}{2304}=\frac{4}{9}$ *Ans.*

qt. pt. pt. bu. qt. pt. pt.
(12) 2 $1\frac{1}{3}=\frac{16}{3}$: 1 1 $1\frac{2}{3}=\frac{203}{3}$. *Ans.* $\frac{16}{203}$.

yd. ft. in. in. yd. ft. in. in.
(13) 1 1 $1\frac{9}{11}=\frac{548}{11}=\frac{3836}{77}$: 3 2 $8\frac{6}{7}=\frac{986}{7}=\frac{10846}{77}$.
$\frac{3836}{10846}=\frac{1918}{5423}$ *Ans.*

ARTICLE 164.

(2)

	hr.	min.
$\frac{2}{3}$ da.=	16	0
$\frac{3}{4}$ hr.=		45
Ans.	16	45

(3)

	da.	hr.	min.
$\frac{1}{4}$ wk.=	1	18	0
$\frac{1}{4}$ da. =		6	0
$\frac{1}{4}$ hr. =			15
Ans.	2	0	15

(4)

	da.	hr.	min.	sec.
$\frac{2}{3}$ wk. =	4	16	0	0
$\frac{5}{9}$ da. =		13	20	0
$\frac{2}{3}$ hr. =			40	0
$\frac{2}{3}$ min.=				40
Ans.	5	6	00	40

(5)

	gal.	qt.	pt.	gi.
$\frac{11}{12}$ gal. =		3	1	$1\frac{1}{3}$
$\frac{1}{12}$ hhd. =	5	1	0	0
Ans.	6	0	1	$1\frac{1}{3}$

(6)

	A.	R.	P.
$2\frac{6}{7}$ R.=		2	$34\frac{2}{7}$
$\frac{2}{3}$ A.=		2	$26\frac{2}{3}$
		3.	$28\frac{4}{7}$
Ans.	2	1	$9\frac{11}{21}$

(7)

	hr.	min.	sec.
$\frac{7}{9}$ da.=	18	40	0
$\frac{1}{18}$ hr.=		3	20
Ans.	18	36	40

(8)

	cts.
$\frac{5}{8}$ \$ =	$62\frac{1}{2}$
$\frac{3}{4}$ d.=	$7\frac{1}{2}$
Ans.	55

(9)

	pwt.	gr.
$\frac{3}{5}$ oz. =	12	0
$\frac{7}{8}$ pwt.=		21
Ans.	11	3

(10)

	fur.	rd.	ft.	in.
$\frac{2}{9}$ mi.=	1	31	1	10
$\frac{7}{11}$ fur.=		25	7	6
Ans.	1	5	$10\frac{1}{2}$	4
Or,	1	5	10	10

(11)

	hr.	min.	sec.
$\frac{6}{7}$ da.=	20	34	$17\frac{1}{7}$
$\frac{2}{3}$ hr.=		40	0
Ans.	19	54	$17\frac{1}{7}$

(12)

	qr.	na.
$\frac{7}{8}$ E.E.=	4	$1\frac{1}{2}$
$\frac{2}{7}$ yd. =	1	$0\frac{4}{7}$
Ans.	3	$0\frac{13}{14}$

ARTICLE 165.—PROMISCUOUS EXAMPLES.

(1) $\frac{32989}{56981}=\frac{2999\times11}{2999\times19}=\frac{11}{19}$ *Ans.*

(2) $\frac{3}{4}+\frac{8}{9}=\frac{27}{36}+\frac{32}{36}=\frac{59}{36}=1\frac{23}{36}$: $\frac{5}{8}+\frac{2}{7}=\frac{35}{56}+\frac{16}{56}=\frac{51}{56}$: $\frac{3}{10}+\frac{11}{15}=\frac{9}{30}+\frac{22}{30}=\frac{31}{30}=1\frac{1}{30}$: $2+3=5$, $\frac{1}{2}+\frac{2}{3}+\frac{5}{14}+\frac{8}{21}=\frac{21}{42}+\frac{28}{42}+\frac{15}{42}+\frac{16}{42}=\frac{80}{42}=\frac{40}{21}=1\frac{19}{21}$, $5+1\frac{19}{21}=6\frac{19}{21}$ *Ans.*

(3) $\frac{3}{4}-\frac{2}{3}=\frac{9}{12}-\frac{8}{12}=\frac{1}{12}$: $\frac{1}{2}-\frac{4}{11}=\frac{11}{22}-\frac{8}{22}=\frac{3}{22}$: $3\frac{4}{7}-1\frac{4}{5}=\frac{25}{7}-\frac{9}{5}=\frac{125}{35}-\frac{63}{35}=\frac{62}{35}=1\frac{27}{35}$: $3\frac{5}{8}=\frac{29}{8}$, $\frac{1}{3}$ of $3\frac{1}{2}=\frac{1}{3}$ of $\frac{7}{2}=\frac{7}{6}$, $\frac{29}{8}-\frac{7}{6}=\frac{87}{24}-\frac{28}{24}=\frac{59}{24}=2\frac{11}{24}$ *Ans.*

(4) $\frac{5}{9}$ of $\frac{7}{10}=\frac{7}{18}$: $\frac{2}{5}$ of $\frac{7}{12}=\frac{7}{30}$: $\frac{7}{18}+\frac{7}{30}=\frac{35}{90}+\frac{21}{90}=\frac{56}{90}=\frac{28}{45}$ *Ans.*

(5) $1\frac{3}{4}\div2\frac{1}{2}=\frac{7}{4}\times\frac{2}{5}=\frac{7}{10}$; $5\frac{1}{2}\div3\frac{1}{8}=\frac{11}{2}\times\frac{8}{25}=\frac{44}{25}$; $\frac{7}{10}+\frac{44}{25}=\frac{35}{50}+\frac{88}{50}=\frac{123}{50}=2\frac{23}{50}$ *Ans.*

(6) $10\times\frac{3}{5}=\frac{30}{5}=6$ *Ans.* **(7)** $10\div\frac{3}{5}=10\times\frac{5}{3}=\frac{50}{3}=16\frac{2}{3}$ *Ans.*

(8) If $\frac{3}{7}$ of any thing are taken from it, there will be $\frac{4}{7}$ left; if 16 is 4-*sevenths*, 1-*fourth* of 16, which is 4, is 1-*seventh*, and if 4 is 1-seventh, 7×4=28, is the number.

(9) If to any thing there be added $\frac{3}{7}$ of it, the sum will be $\frac{7}{7}+\frac{3}{7}=\frac{10}{7}$; if 20 is 10-*sevenths*, 1-*tenth* of 20, which is 2, is 1-*seventh*; and if 2 is 1-seventh, 7 times 2, which are 14, is the number.

(10) $\frac{1}{3}$ of $\frac{5}{8}=\frac{5}{24}$, and $\frac{5}{8}-\frac{5}{24}=\frac{15}{24}-\frac{5}{24}=\frac{10}{24}=\frac{5}{12}$ part left. Or, the part left may be found thus. If he sell $\frac{1}{3}$ of his share, he has $\frac{2}{3}$ of it left, and $\frac{2}{3}$ of $\frac{5}{8}=\frac{10}{24}=\frac{5}{12}$.
$\frac{5}{12}$ of \$900$=\frac{4500}{12}=$\$375 *Ans.*

(11) $\frac{1}{3}$ of $\frac{7}{12}=\frac{7}{36}$, $1944\frac{4}{9}=\frac{17500}{9}$.
Having 7-thirty-sixths, if we divide by 7, we get 1-thirty-sixth; then, multiplying this by 36, we get the value of the whole. Hence the answer will be found by multiplying the $\frac{7}{36}$ of the value by $\frac{36}{7}$. $\frac{17500}{9}\times\frac{36}{7}$
$=\$10000$ *Ans.*

(12) $\frac{2}{3}$ of $2=\frac{4}{3}$; $3=\frac{9}{3}$: *Ans.* $\frac{4}{9}$

(13) $\frac{176}{368}=\dfrac{16\times11}{16\times23}=\frac{11}{23}$ *Ans.*

(14) $\frac{1}{8}+\frac{1}{18}+\frac{13}{111}=\frac{333}{2664}+\frac{148}{2664}+\frac{312}{2664}=\frac{793}{2664}$;
$\frac{25}{37}-\frac{793}{2664}=\frac{1800}{2664}-\frac{793}{2664}=\frac{1007}{2664}$ *Ans.*

(15) $4\frac{9}{14}=\frac{65}{14}$; $\frac{3}{10}$ of $\frac{7}{12}$ of $\frac{65}{14}=\dfrac{1\times1\times13}{2\times4\times2}=\frac{13}{16}$; $1-\frac{13}{16}=\frac{3}{16}$
[*Ans.*

(16) $\frac{2}{3}\div\frac{5}{7}=\frac{2}{3}\times\frac{7}{5}=\frac{14}{15}$; $\frac{5}{8}\div\frac{10}{11}=\frac{5}{8}\times\frac{11}{10}=\frac{11}{16}$;
$\frac{14}{15}-\frac{11}{16}=\frac{224}{240}-\frac{165}{240}=\frac{59}{240}$ *Ans.*

(17) If a man walk 2044 rods in 7-fifteenths of an hour, he will walk 1-seventh of 2044 rods in 1-fifteenth of an hour, and 15 times this distance in an hour; this, multiplied by $1\frac{14}{15}$, will give the distance walked in $1\frac{14}{15}$ hours. Thus, $\frac{1}{7}$ of $2044=292$, and $292\times15=4380=$rods walked in an hour; and $4380\times1\frac{14}{15}=\frac{4380}{1}\times\frac{29}{15}=8468$ rods *Ans.*

Or, $\frac{2044}{1}\times\frac{15}{7}\times\frac{29}{15}=\dfrac{2044\times29}{7}=\dfrac{292\times29}{1}=8468$ *Ans.*

(18) $1\frac{1}{4}$ feet$=15$ inches$=\frac{45}{3}$: $3\frac{1}{3}=\frac{10}{3}$: $\frac{10}{45}=\frac{2}{9}$ *Ans.*

(19) $3\frac{1}{5}+3\frac{2}{3}=\frac{16}{5}+\frac{11}{3}=\frac{48}{15}+\frac{55}{15}=\frac{103}{15}$ *Ans.* $\frac{48}{103}$ and $\frac{55}{103}$.

$\$$ $\$$ $\$$ $\$$ $\$$

(20) $\frac{5}{8}$ of $2400=5\times300=1500$: $1500+500=2000$:
If $\frac{5}{4}$ of B's money is \$2000, $\frac{1}{4}$ is $\frac{1}{5}$ of \$2000, which is \$400, and the whole will be 4 times \$400, which is \$1600 *Ans.*

(21) Since 2200 is five-twelfths of what the elder son received, one-fifth of this will be one-twelfth of what he received; and twelve times this last sum will be the whole of the elder son's share:

$\frac{1}{5}$ of $\frac{2200}{1}$=440, and 440×12=5280=elder son's share; for a similar reason, dividing this last sum by 16, and multiplying by 35, will give the whole estate: $\frac{1}{16}$ of 5280=330, and 330×35=11550, whole estate:

$$\begin{array}{lrr}
\text{Youngest son's share} & 2200 & 11550 \\
\text{Eldest son's share . .} & \underline{5280} & \underline{7480} \\
 & 7480 & 3)\underline{4070} \\
\end{array}$$

Each daughter's share=1356\frac{2}{3}$

DECIMAL FRACTIONS.

Suggestions to Teachers.—As it is impossible for pupils to understand the primary rules without first becoming well acquainted with Numeration and Notation, so in studying decimals it is equally important to become familiar with their numeration and notation; pupils should, therefore, be drilled at the black-board until they can read and write decimal fractions with ease and accuracy.

Article **175.**—Numeration.

To be read.—1st Column. (1) Five *tenths.* (2) Six *hundredths.* (3) Three *thousandths.* (4) Twenty-eight *thousandths.* (5) Three hundred and forty-one *thousandths.*

2d Column. (1) Three *ten-thousandths.* (2) Six hundred and twenty-five *ten-thousandths.* (3) Two thousand three hundred and seventy-four *ten-thousandths.* (4) Two thousand and six *ten-thousandths.* (5) One hundred and four *ten-thousandths.*

3d Column. (1) Four *hundred-thousandths.* (2) One hundred and thirty-seven *hundred-thousandths.* (3) Two thousand three hundred and seventy-six *hundred-thousandths.* (4) Thirty-one thousand four hundred and fifty-six *hundred-thousandths.* (5) One thousand and seven *hundred-thousandths.*

4th Column. (1) Seven *millionths.* (2) One hundred and thirty-three *millionths.* (3) One thousand seven hundred and sixty-eight *millionths.* (4) Forty thousand and thirty-five *millionths.* (5) Three hundred and sixty thousand and four *millionths.*

5th Column. (1) Eight *ten-millionths.* (2) Nine *hundred-millionths.* (3) One million ten thousand one hundred and one *ten-millionths.* (4) One hundred thousand three hundred and four *hundred-millionths.* (5) Forty thousand and five *hundred-millionths.*

6th Column. (1) Six units and five *tenths.* (2) Sixty units and four *hundredths.* (3) One hundred and eighty-four units and one hundred and seventy-three *thousandths.*

7th Column. (1) Six *tenths*, or Six million *ten-millionths.* (2) Eight *thousandths*, or Eighty thousand *ten-millionths.* (3) One hundred and two *thousandths*, or One million and twenty thousand *ten-millionths.*

8th Column. (1) Forty thousand five hundred and four units, and one thousand and thirty-seven *hundred-thousandths.* (2) Fifty-four million units, and fifty-four *millionths.* (3) Thirty million seven hundred and one thousand units, and one million thirty-seven thousand and twenty-five *ten-millionths.*

ARTICLE 176.

(1) $\frac{5}{10}=.5$; $\frac{5}{100}=.05$; $\frac{135}{10000}=.0135$; $\frac{203}{100000}=.00203$; $\frac{4006}{10000000}=.0004006$

(2) $13\frac{7}{10}=13.7$; $24\frac{8}{100}=24.08$; $30\frac{25}{1000}=30.025$; $6\frac{4}{10000}=6.0004$; $8\frac{105}{1000000}=8.000105$

(3) .4
(4) .26
(5) .35
(6) .08
(7) .005
(8) .304
(9) .4125
(10) .0205
(11) .0008
(12) .20304
(13) .00605
(14) .00009
(15) .300004
(16) .000203
(17) .000007
(18) .0000024
(19) .0080006
(20) .00000002
(21) .00000907
(22) .20020003
(23) .01010001
(24) 20.25
(25) 106.037
(26) 1000.001
(27) 200.025
(28) 29.000029
(29) .001000005
(30) 200.0000000002
(31) 65.006005

ARTICLE 177.

(32) 14.3
(33) 32.04
(34) 1.0502

ARTICLE 178.—ADDITION.

(2) 3.25
6.4
.35
Ans. 10.00

(3) 21.611
6888.32
3.4167
Ans. 6913.3477

(4) 6.61
636.1
6516.14
67.1234
5.1233
Ans. 7231.0967

(5) 4.8
43.31
74.019
11.204
Ans. 133.333

(6) 45.019
7.00071
93.4327
6.0401
Ans. 151.49251

(7) 432.432
61.0793
100.07794
6.009
1000.1001
Ans. 1599.69834

(8) 16.041
9.000094
33.27
8.969
32.719906
Ans. 100.000000

(9) 204.0009
103.00000009
42.009099
430.99
220.0000009
Ans. 999.99999999

(10) .0035
.00035
.000035
.0000035
Ans. .0038885

ARTICLE 179.—SUBTRACTION.

(2) 5.03
2.115
Ans. 2.915

(3) 24.0042
13.7013
Ans. 10.3029

(4) 170.0035
68.00181
Ans. 102.00169

(5) .0142
.005
Ans. .0092

(6) .05
.0024
Ans. .0476

(7) 13.5
8.037
Ans. 5.463

(8) 3.00000
.00003
Ans. 2.99997

(9) 29.0029
19.003
Ans. 9.9999

(10) 5.000
.125
Ans. 4.875

(11) 10000.0000
.0001
Ans. 9999.9999

(12) 1.000000
.000001
Ans. .999999

(13) .025
.000025
Ans. .024975

ARTICLE 181.—MULTIPLICATION.

(2) .35
7
Ans. 2.45

(5) $.15\times.7=\frac{15}{100}\times\frac{7}{10}=\frac{105}{1000}=.105$ *Ans.*

(8) $1.035\times17=17.595$ *Ans.*

(9) $$\begin{array}{r} 19 \\ .125 \\ \hline 95 \\ 38 \\ 19 \\ \hline 2.375 \end{array}$$
Ans. 2.375

(10) $$\begin{array}{r} 4.5 \\ 4 \\ \hline 18.0 \end{array}$$
Ans. 18.0

(11) $$\begin{array}{r} .625 \\ 64 \\ \hline 2500 \\ 3750 \\ \hline 40.000 \end{array}$$
Ans. 40.000

(12) $$\begin{array}{r} 61.76 \\ .0071 \\ \hline 6176 \\ 43232 \\ \hline .438496 \end{array}$$
Ans. .438496

(13) $$\begin{array}{r} 1.325 \\ .0716 \\ \hline 7950 \\ 1325 \\ 9275 \\ \hline .0948700 \end{array}$$
Ans. .0948700

(14) $$\begin{array}{r} 79000 \\ .079 \\ \hline 711000 \\ 553000 \\ \hline 6241.000 \end{array}$$
Ans. 6241.000

(15) $$\begin{array}{r} .1 \\ .01 \\ \hline .001 \end{array}$$
Ans. .001

(16) $$\begin{array}{r} 1. \\ .0001 \\ \hline .0001 \end{array}$$
Ans. .0001

(17) $$\begin{array}{r} .043 \\ .0021 \\ \hline 43 \\ 86 \\ \hline .0000903 \end{array}$$
Ans. .0000903

(18) $$\begin{array}{r} 40000 \\ .000001 \\ \hline .040000 \end{array}$$
Ans. .040000

(19) $$\begin{array}{r} .09375 \\ .1000064 \\ \hline 37500 \\ 56250 \\ 9375 \\ \hline .09375600000 \end{array}$$
Ans. .09375600000

DIVISION.

Suggestions to Teachers.—The division of decimals is generally a troublesome subject to pupils; this arises from a want of attention to the rule. Should the pupil be at a loss to understand why, in some cases, when the divisor and dividend are both decimals, the quotient should be a whole number, let him read the remarks on the division of fractions, page 79. When the divisor contains more decimal places than the dividend, it is best, before commencing the division, to reduce them both to the same denomination, that is, to make the number of decimal places the same in both; the quotient will then be a whole number.

Article 183.

(5) $\frac{1.125}{.03}=37.5$ *Ans.*

(6) $\frac{2}{.008}=\frac{2.000}{.008}=250$ *Ans.*

(7) $\frac{37.2}{5.}=\frac{37.20}{5.}=7.44$ *Ans.* **(8)** $\frac{86.075}{27.5}=3.13$ *Ans.*

(9) $\frac{24.73704}{3.44}=7.191$ *Ans.* **(10)** $\frac{206.166492}{4.123}=50.004$ *Ans.*

(11) $\frac{100.8788}{454}=.2222$ *Ans.* **(12)** $\frac{.000343}{3.43}=.0001$ *Ans.*

(13) $\frac{9811.0047}{.108649}=\frac{9811.004700}{.108649}=90300$ *Ans.*

(14) $\frac{.21318}{.19}=1.122$ *Ans.*

(15) $\frac{102048}{.3189}=\frac{102048.0000}{.3189}=320000$ *Ans.*

(16) $\frac{.102048}{3189}=.000032$ *Ans.* **(17)** $\frac{9.9}{.0225}=\frac{9.9000}{.0225}=440$ *Ans.*

(18) 10).10
Ans. .01

(19) .1)1.0
Ans. 10.

(20) .01)10.00
Ans. 1000

(21) $\frac{1.7}{64}=\frac{1.7000000}{64}=.0265625$ *Ans.*

(22) 80).080
Ans. .001

(23) 1000).001000
Ans. .000001

(24) .001).001
Ans. 1.

(25) .0000001).0001000
Ans. 1000.

(26) .000004).010000
Ans. 2500.

(27) $\frac{1.5}{.7}=\frac{1.5000000}{.7}=2.142857+$

ARTICLE 185.—CASE I.

(2) $\frac{1}{8}=\frac{1.000}{8}=.125$ *Ans.* **(3)** $\frac{4}{5}=\frac{4.0}{5}=.8$ *Ans.*

(4) $\frac{4}{25}=\frac{4.00}{25}=.16$ *Ans.* **(5)** $\frac{3}{40}=\frac{3.000}{40}=.075$ *Ans.*

(6) $\frac{15}{16}=\frac{15.0000}{16}=.9375$ *Ans.* **(7)** $\frac{1}{1250}=\frac{1.0000}{1250}=.0008$ *Ans.*

(8) $\frac{9}{400}=\frac{9.0000}{400}=.0225$ *Ans.*

(9) $\frac{1}{256}=\frac{1.00000000}{256}=.00390625$ *Ans.*

(10) $\frac{5}{6}=\frac{5.0000}{6}=.8333+$ *Ans.*

(11) $\frac{4}{33}=\frac{4.000000}{33}=.121212+$ *Ans.*

(12) $\frac{1}{11}=\frac{1.000000}{11}=.090909+$ *Ans.*

ARTICLE 186.—CASE II.

(2) $.6=\frac{6}{10}=\frac{3}{5}$ *Ans.* **(3)** $.25=\frac{25}{100}=\frac{1}{4}$ *Ans.*

(4) $.375=\frac{375}{1000}=\frac{3}{8}$ *Ans.* **(5)** $4.02=4\frac{2}{100}=4\frac{1}{50}$ *Ans.*

(6) $8.415=8\frac{415}{1000}=8\frac{83}{200}$ *Ans.* **(7)** $.033=\frac{33}{1000}$ *Ans.*

(8) $.5625=\frac{5625}{10000}=\frac{9}{16}$ *Ans.* **(9)** $.34375=\frac{34375}{100000}=\frac{11}{32}$ *Ans.*

(10) $.1484375=\frac{1484375}{10000000}=\frac{19}{128}$ *Ans.*

ARTICLE 187.—CASE III.

(3) .125 bu. $=.125\times4=.5$ pk. *Ans.*

(4) .7 pk. $=.7\div4=.175$ bu. *Ans.*

(5) .0625 lb. $\times12=.75$ oz. *Ans.*

(6) .05 yd. $\times4\times4=.8$ na.

(7) .00546875 A. $\times4\times40=.875$ P. *Ans.*

(8) .0004375 mi.×8×40=.14 rd. *Ans.*

(9) .25 pt.÷2=.125 qt. : .125 qt.÷4=.03125 gal. *Ans.*

(10) .6 pt.÷2=.3 qt. : .3 qt.÷8=.0375 pk. :
.0375 pk.÷4=.009375 bu. *Ans.*

(11) .3 min.÷60=.005 hr.: .005 hr.÷24=.0002083 da.+ *Ans.*

(12) .7 rd.÷40=.0175 fur : .0175 fur.÷8=.0021875 mi. *Ans.*

ARTICLE 188.—CASE IV.

(2) .875 yd.×4=3.5 qr. : .5 qr.×4=2. na. *Ans.* 3 qr. 2 na.

(3) .7 lb.×12=8.4 oz. : .4 oz.×20=8. pwt. *Ans.* 8 oz. 8 pwt.

(4) .8125 bu.×4=3.25 pk. : .25 pk.×8=2. qt.
Ans. 3 pk. 2 qt.

(5) .3375 A.×4=1.35 R. : .35 R.×40=14. P. *Ans.* 1 R. 14 P.

(6) .04318 mi.×8=.34544 fur. : .34544 fur.×40
=13.8176 rd. : .8176 rd.×$5\frac{1}{2}$=4.4968 yd. : .4968 yd.×3
=1.4904 ft. : .4904 ft.×12=5.8848 in.
Ans. 13 rd. 4 yd. 1 ft. 5.8848 in.

(7) .33625 cwt.×4=1.345 qr. : .345 qr.×25=8.625 lb. :
.625 lb.×16=10 oz. *Ans.* 1 qr. 8 lb. 10 oz.

ARTICLE 189.—CASE V.

(2) 2 pk. 4 qt.=20 qt. : 1 bu.=32 qt. : $\frac{20}{32}=\frac{5}{8}$=.625 *Ans.*

(3) 13 hr. 30 min.=810 min. : 1 da.=1440 min.
$\frac{810}{1440}=\frac{9}{16}$=.5625 *Ans.*

(4) 1 lb.=256 dr. : $\frac{9}{256}$=.03515625 *Ans.*

(5) 1 A.=160 P $\frac{.028}{160}$=.000175 *Ans.*

(6) 1 da.=1440 min. $\frac{7}{1440}$=.0048611+ *Ans.*

(7) 4 gal. 1 qt. 1.28 pt.=35.28 pt. : 1 hhd.=504 pt.
$\frac{35.28}{504}$=.07 *Ans.*

(8) 3 pk. 7 qt. 1 pt.=63 pt. : 2 bu. 2 pk. 4 qt.=168 pt.
$\frac{63}{168}=\frac{3}{8}=.375$ *Ans.*

(9) 99 pages=$\frac{99}{512}$ of 512 pages : $\frac{99}{512}=.193359375$ *Ans.*

(10) 55 A. 2 R. 17 P.=8897 P.
229 A. 2 R. 16 P.=36736 P. $\frac{8897}{36736}=.2421875$ *Ans.*

ARTICLE 190.

(1) $0.4×9=$3.6 : $0.1875×12=$2.25 :
$3.6+$2.25=$5.85 *Ans.*

(2) $0.45×2.3=$1.035 : $0.375×1.5=$.5625 :
$1.035+$.5625=$1.5975 *Ans.*

(3) $2.6875×16=$43 : $2.6875×16¼=$43.671875 *Ans.*

(4) $35.25÷$0.75=47 bu. *Ans.*

(5) $98.4÷$2.5625=38.4 yd. *Ans.*

(6) 6 cwt. 2 qr.=6.5 cwt. : $3.25×6.5=$21.125 *Ans.*

(7) 7 hhd. 23 gal.=7.365079+
$49×7.365079=$360.8888+ *Ans.*

(8) 343 yd. 3 qr.=343.75 yd. : $0.16×343.75=$55 *Ans.*

(9) 14 bu. 3 pk. 4 qt.=14.875 bu.
$.625×14.857=$9.296875 *Ans.*

(10) 13 A. 2 R. 35 P.=13.71875 A.
$17.28×13.71875=$237.06 *Ans.*

(11) $19.065÷$1.24=15.375 yd.=15 yd. 1 qr. 2 na. *Ans.*

(12) $9.296875÷$.3125=29.75 bu.=29 bu. 3 pk. *Ans.*

(13) $59.265÷$4.32=13.71875 A.=13 A. 2 R. 35 P. *Ans.*

(14) .34 yd.×4=1.36 qr. : 1.36 qr.+.325 qr.=1.685 qr. :
.685 qr.×4=2.74 na. : 2.74 na.+.4 na.=3.14 na.
Ans. 1 qr. 3.14 na.

(15)

	yd.	qr.	na.
1.53 yd.=	1	2	0.48
1.32 qr.=		1	1.28
Ans.	1	0	3.20

(16) 365.25×.05=18.2625 da. .2625 da.×24=6.3 hr.
6.3 hr.—.5 hr.=5.8 hr. .8 hr.×60=48. min.
Ans. 18 da. 5 hr. 48 min.

(17) .41 da.=9.84 hr. : 9.84 hr.—.16 hr.=9.68 hr. :
.68 hr.×60=40.8 min. : .8 min.×60=48. sec.
Ans. 9 hr. 40 min. 48 sec.

(18) .4 T.×4=1.6 hhd. : 1.6 hhd.+.3 hhd.=1.9 hhd. :
1.9×63=119.7 gal. : 119.7 gal.+.8 gal.=120.5 gal. :
120.5 gal.×4=482 qt.=964 pt. *Ans.*

(19) 365.25 da.×.3=109.575 da. : .575 da.×24=13.8 hr. :
.8 hr.×60=48 min. *Ans.* 109 da. 13 hr. 48 min.

(20) 365.25 da.×.005×24×60×60=157788 sec. *Ans.*

(21) 1 cord=128×1728=221184 cu. in.
$\frac{1}{221184}$=.000004+ *Ans.*

(22) 17 mi. 3 fur. 15 rd.=17.421875 mi. :
$690.35×17.421875=$12027. 19140625 *Ans.*

Remarks.—Pupils must have a thorough knowledge of common fractions, before they can understand fully the reason of the rules in decimals.

When a pupil is in doubt with regard to the accuracy of the result in any operation involving decimals, let him convert the decimals into common fractions, and then perform the work; the results, in both cases, ought to be the same.

It is a useful exercise to perform the same operations in equivalent common and decimal fractions. Thus, they may be required to perform the operations indicated in the following examples, by the rules for common fractions; then to convert the common fractions into decimals, and work by the rules for decimals.

EXAMPLES.

	By common fractions.	By decimals.
(1) $\frac{1}{4}+\frac{3}{8}$.	$\frac{1}{4}+\frac{3}{8}=\frac{2}{8}+\frac{3}{8}=\frac{5}{8}$ *Ans.*	$\frac{1}{4}$=.25 $\frac{3}{8}$=.375 $\frac{5}{8}$=.625 *Ans.*

(2) $\frac{1}{8}+\frac{3}{16}$. *Ans.* $\frac{5}{16}=.3125$

(3) $\frac{1}{2}-\frac{1}{8}$. *Ans.* $\frac{3}{8}=.375$

(4) $\frac{1}{4}\times\frac{3}{8}$. *Ans.* $\frac{3}{32}=.09375$

(5) $\frac{7}{20}\div\frac{1}{4}$. *Ans.* $1\frac{2}{5}=1.4$

(6) $\frac{27}{500}\div\frac{3}{4}$. *Ans.* $\frac{9}{125}=.072$

The great advantage of decimals is, that the operations are performed with the same facility as in whole numbers; and, that the most complicated arithmetical expressions may be reduced to equivalent decimals that are comparatively simple.

PROPORTION.

Suggestions to Teachers.—Simple proportion is justly regarded as one of the most important subjects in Arithmetic; particular care should be taken, therefore, to render the pupil well acquainted with its principles. First, let him thoroughly understand the meaning of ratio; if the questions and exercises in the Arithmetic are not sufficient, let the instructor add such as he may deem necessary. It should be kept before the pupil that there is no such thing as ratio between quantities of different kinds, and hence, that the first and second terms in every proportion must be of the same kind.

In performing the work necessary to obtain the fourth term, it may be asked, do we not use a concrete number as a multiplier, and, also, as a divisor? The answer is, we do not; we multiply and divide by abstract numbers that represent the ratio of the first term to the second. Thus, the question, if 2 yards of cloth cost 5 dollars, how many dollars will 6 yards cost? being stated, is,

yd. yd. $
As 2 : 6 : : 5 to the answer.

In performing the operation, we multiply the 5 dollars by 6, and divide by 2; because, the ratio of 2 to 6, is the same as that of 2 yards to 6 yards. The same method of reasoning will apply to every case, both in simple and compound proportion.

Article 203.

(8) As 4 hats : 10 hats : : $14 : $35 *Ans.*

(9) As 3 caps : 11 caps : : 69 cts. : $2.53 *Ans.*

(10) As 4 yd. : 9 yd. : : $7 : $15.75 *Ans.*

(11) As 8 yd. : 12 yd. : : $32 : $48 *Ans.*

(12) As 12 yd. : 8 yd. : : $48 : $32 *Ans.*

(13) As $32 : $48 : : 8 yd. : 12 yd. *Ans.*

(14) As $48 : $32 : : 12 yd. : 8 yd. *Ans.*

(16) As 12 men : 8 men : : 24 da. : 16 da. *Ans.*

(17) As 2 da. : 8 da. : : 60 men : 240 men *Ans.*

(19) As 6 gal. : 2 hhd. : : 65 cts. *Ans.*
Or, 6 gal. : 2×63 gal. : : 65 cts.
1 gal. : 21 gal. : : 65 cts. : $13.65 *Ans.*

(20) As 5 cwt. 3 qr. 10 lb. : 35 cwt. 1 qr. : : $21.06 : $126.90 *Ans.*

(21) As 1 yd. 2. qr. : 1 qr. 2 na. : : $2.50 : $0.625 *Ans.*

(22) As 90 bu. : 450 bu. : : 6 da. : 30 da. *Ans.*

(23) As 5 da. : 15 da. : : 6 men : 18 men *Ans.*

(24) As 60 bu. p. : 140 bu. p. : : 15 bu. c. : 35 bu. *Ans.*

(25) As 3 cwt. 1 qr. : 16 cwt. 1 qr. : : $22.60 : $113.00 *Ans.*

(26) As 4 ft. 6 in. in shadow : 180 ft. shadow : : 3 ft. : 120 ft. *Ans.*

(27) As 12 hr. : 9 hr. : : 60 da. : 45 da. *Ans.*

(28) As $1 : $2200 : : $.60 : $1320 A's, *Ans.*
As $1 : $1800 : : $.60 : $1080 B's, *Ans.*

(29) $800.30+$250+375.10+$500+$115=$2040.40
As $2040.40 : $612.12 : : $1 : $.30 *Ans.* 30 cts.

(30) As $6 : $8 : : 9 oz. : 12 oz. *Ans.*

(31) As $300 : $250 : : 6 mon. : 5 mon. *Ans.*

(32) 27×7=189, 36—27=9.
As 9 mi. : 189 mi. : : 1 da. : 21 da. *Ans.*

(33) 9 hr. : 12 hr. : : $\$15\frac{2}{3}$: $\$20.88\frac{8}{9}$, one month's services when he works 12 hr. a day. $\$20.88\frac{8}{9} \times 4\frac{2}{5} = \$91.91\frac{1}{9}$ *Ans.*

(34) As 5 lb. : $\frac{3}{4}$ lb. : : \$$\frac{5}{8}$: \$$\frac{3}{32}$ *Ans.*

(35) As 6 yd. : $7\frac{3}{8}$ yd. : : \$$5\frac{3}{5}$: \$$6\frac{53}{60}$ *Ans.*

(36) As $\frac{1}{3}$ bu. : $\frac{1}{2}$ bu. : : \$$\frac{3}{8}$: \$$\frac{9}{16}$. *Ans.* $\frac{3}{1}\times\frac{1}{2}\times\frac{3}{8}=\frac{9}{16}$.

(37) As $1\frac{3}{4}$ yd. : 2 yd. : : \$$\frac{7}{24}$: \$$\frac{1}{3}$. *Ans.* $\frac{4}{7}\times\frac{2}{1}\times\frac{7}{24}=\frac{1}{3}$.

(38) As \$$29\frac{3}{4}$: $31\frac{1}{4}$: : $59\frac{1}{2}$ yd. : *Ans.*
$\frac{4}{119}\times\frac{125}{4}\times\frac{119}{2}=\frac{125}{2}=62\frac{1}{2}$ yd. *Ans.*

(39) As .85 gal. : .25 gal. : : \$1.36 : \$0.40 *Ans.*

(40) As 61.3 lb. : 1.07 lb. : : \$44.9942 : \$0.78538 *Ans.*

(41) As $\frac{5}{7}$ yd. : $\frac{9}{11}$ E. En. : : \$$\frac{3}{5}$: *Ans.*
$\frac{20}{7}$ qr. : $\frac{45}{11}$ qr. : : \$$\frac{3}{5}$: *Ans.* $\frac{7}{20}\times\frac{45}{11}\times\frac{3}{5}=\$\frac{189}{220}$ *Ans.*

(42) As $\frac{3}{7}$ yd. : $17\frac{3}{8}$ yd. : : \$$4\frac{2}{5}$: *Ans.*
$\frac{7}{3}\times\frac{139}{8}\times\frac{22}{5}=\$178.38\frac{1}{3}$ *Ans.*

cogs. cogs. rev. rev.
(43) As 26 : 35 : : 1 : $1\frac{9}{26}$. Hence, the smaller wheel gains $\frac{9}{26}$ of a revolution in each revolution of the larger wheel.

rev. rev. rev. of larger.
Then, $\frac{9}{26}$: 10 : : 1 : $28\frac{8}{9}$=revolution of larger, *Ans.*

(44) 1 gal. =32 gills, 32—1=31.
As 32 : 31 : : 100 gal. : $96\frac{7}{8}$ gal. *Ans.*

(45) As 70 p. : 20 p. : : 60 sec. : $17\frac{1}{7}$ sec. *Ans.*
$1142\times17\frac{1}{7}=19577\frac{1}{7}$ ft.=3 mi. 5 fur. 145 yd. $2\frac{1}{7}$ ft. *Ans.*

(46) As 25 ft. : 25 ft. 5.25 in. : : 643 ft. 8 in. :
654 ft. 11.17 in. *Ans.*

(47) As 60 lb. : 2136 lb. : : 75 cts. : \$26.70 *Ans.*

(48) As 60 lb. : 1225 lb. : : 81 cts. : \$16.53+ *Ans.*

(49) As 56 lb. : 1000 lb. : : 63 cts. : \$11.25 *Ans.*

(50) As 32 lb. : 3000 lb. : : 24 cts. : \$22.50 *Ans.*

ARTICLE 205.

(2) $\left.\begin{array}{l}2 \text{ da.} : 10 \text{ da.}\\ 4 \text{ hr.} : 8 \text{ hr.}\end{array}\right\}$: : 24 mi. : 240 mi. *Ans.*

SUGGESTION TO TEACHERS.—In stating questions in Compound Proportion, it will be found both interesting and instructive for the pupil to give the reason for the arrangement of each pair of terms, as in the following statements.

NOTE.—For convenience, the third term is placed below the second.

(4) As 18 rd. : 72 rd. The more rods, the more men.
8 da. : 12 da. The less days, the more men.
: : 16 men. *Ans.* 96 men.

This statement is read thus: As 18 rods are to 72 rods, and as 8 days are to 12 days; so are 16 men to the required number of men.

(5) As 6 p. : 15 p. The more persons, the more dollars.
8 mon. : 20 mon. The more months, the more dollars.
: : $150. *Ans.* $937.50.

(6) As 7 da. : 9 da. The more days, the more miles.
6 hr. : 11 hr. The more hours, the more miles.
: : 217 mi. *Ans.* $511\frac{1}{2}$ mi.

(7) As $100 : $75. The less dollars, the less interest.
12 mon. : 9 mon. The less months, the less interest.
: : $6. *Ans.* $3.375

(8) As 10100 lb. : 100 lb. The more lb., the less miles.
20 cts. : $60.60 The more money, the more mi.
: : 20 mi. *Ans.* 60 mi.

(9) As 12 cwt. 3 qr. : 10 T. The more weight, the more money.
400 mi. : 75 mi. The less miles, the less money.
: : $57.12 *Ans.* $168.

(10) As 20 men : 18 men. The more men, the less days.
40 rd. l. : 87 rd. l. The more length, the more days.
5 ft. h. : 8 ft. h. The more hight, the more days.
4 ft. t. : 5 ft. t. The more thickness, the more da.
: : 15 days. *Ans.* $58\frac{29}{40}$ days.

(11) As 100 men : 180 men. The less men, the more days.
200 yd. l. : 180 yd. l. The less length, the less days.
3 yd. w. : 4 yd. w. The more width, the more days.
2 yd. d. : 3 yd. d. The more depth, the more days.
8 hr. : 10 hr. The less hours, the more days.
: : 6 days. *Ans.* 24.3 days.

ALIQUOTS OR PRACTICE.

ARTICLE 207.—CASE I.

(3)

	$24=cost of 24 yd. at $1
50 $=\frac{1}{2}$	12=cost of 24 yd. at 50 cts.
$12\frac{1}{2}$—$\frac{1}{4}$	3= " " " " at $12\frac{1}{2}$ cts.
	$15 *Ans.*

(5)

	$173
25=$\frac{1}{4}$	$ 43.25 *Ans.*

(6)

	$45
25 $=\frac{1}{4}$	11.25
$6\frac{1}{4}=\frac{1}{4}$	$2.81\frac{1}{4}$
	$$14.06\frac{1}{4}$ *Ans.*

(7)

	$54
25 $=\frac{1}{4}$	13.50
$12\frac{1}{2}=\frac{1}{2}$	6.75
$6\frac{1}{4}=\frac{1}{2}$	$3.37\frac{1}{2}$
	$$23.62\frac{1}{2}$ *Ans.*

(8)

	$32
50 $=\frac{1}{2}$	16
25 $=\frac{1}{2}$	8
$12\frac{1}{2}=\frac{1}{2}$	4
$6\frac{1}{4}=\frac{1}{2}$	2
Ans.	$30

(10)

	80
	2
	160
25 $=\frac{1}{4}$	20
$12\frac{1}{2}=\frac{1}{2}$	10
Ans.	$190

(11)

	36
	8
	288
50 $=\frac{1}{2}$	18
25 $=\frac{1}{2}$	9
$12\frac{1}{2}=\frac{1}{2}$	4.50
Ans.	$319.50

(12)

	77
	1
	77
50 $=\frac{1}{2}$	38.50
$12\frac{1}{2}=\frac{1}{4}$	$9.62\frac{1}{2}$
Ans.	$$125.12\frac{1}{2}$

(13)

	175
	14
	700
	175
	2450
25 $=\frac{1}{4}$	43.75
$12\frac{1}{2}=\frac{1}{2}$	$21.87\frac{1}{2}$
Ans.	$$2515.62\frac{1}{2}$

(14)

	224
	3
	672
25 $=\frac{1}{4}$	56
$12\frac{1}{2}=\frac{1}{2}$	28
$6\frac{1}{4}=\frac{1}{2}$	14
Ans.	$770

(15)

	462
	1
	462
$6\frac{1}{4}=\frac{1}{16}$	$28.87\frac{1}{2}$
Ans.	$$490.87\frac{1}{2}$

(16)

	185
	1
	185
$33\frac{1}{3}=\frac{1}{3}$	$61.66\frac{2}{3}$
Ans.	$$246.66\frac{2}{3}$

(17)
$$\begin{array}{r|r} & 150 \\ & 3 \\ \hline & 450 \\ 33\frac{1}{3}=\frac{1}{3} & 50 \\ 33\frac{1}{3}=\frac{1}{3} & 50 \\ \hline \textit{Ans.} & \$550 \end{array}$$

(18)
$$\begin{array}{r|r} & 24 \\ & 1 \\ \hline & 24 \\ 16\frac{2}{3}=\frac{1}{6} & 4 \\ \hline \textit{Ans.} & \$28 \end{array}$$

ARTICLE 208.—CASE II.

(2)
$$\begin{array}{r|r} & \$10 \\ & 14 \\ \hline & 140 \\ 2\text{ R.}=\frac{1}{2} & 5 \\ \hline \textit{Ans.} & \$145 \end{array}$$

(3)
$$\begin{array}{r|r} & \$12.50 \\ & 28 \\ \hline & 10000 \\ & 2500 \\ \hline & 350.00 \\ 2\text{ R.}=\frac{1}{2} & 6.25 \\ 1\text{ R.}=\frac{1}{2} & 3.12\frac{1}{2} \\ \hline \textit{Ans.} & \$359.37\frac{1}{2} \end{array}$$

(4)
$$\begin{array}{r|r} & \$12 \\ & 5 \\ \hline & 60 \\ 1\text{ R.}=\frac{1}{4} & 3.00 \\ 10\text{ P.}=\frac{1}{4} & .75 \\ \hline \textit{Ans.} & \$63.75 \end{array}$$

(5)
$$\begin{array}{r|r} & \$18 \\ & 12 \\ \hline & 216 \\ 1\text{ R.}=\frac{1}{4} & 4.50 \\ 10\text{ P.}=\frac{1}{4} & 1.12\frac{1}{2} \\ \hline \textit{Ans.} & \$221.62\frac{1}{2} \end{array}$$

(6)
$$\begin{array}{r|r} & \$12.50 \\ & 14 \\ \hline & 5000 \\ & 1250 \\ \hline & 175.00 \\ 2\text{ R.}=\frac{1}{2} & 6.25 \\ 1\text{ R.}=\frac{1}{2} & 3.12\frac{1}{2} \\ 20\text{ P.}=\frac{1}{2} & 1.56\frac{1}{4} \\ 5\text{ P.}=\frac{1}{4} & .39\frac{1}{16} \\ \hline \textit{Ans.} & \$186.32\frac{13}{16} \end{array}$$

(7)
$$\begin{array}{r|r} & \$1.75 \\ & 3 \\ \hline & 5.25 \\ 2\text{ qr.}=\frac{1}{2} & .87\frac{1}{2} \\ \hline \textit{Ans.} & \$6.12\frac{1}{2} \end{array}$$

(8)
$$\begin{array}{r|r} & \$1.50 \\ & 4 \\ \hline & 6.00 \\ 2\text{ qr.}=\frac{1}{2} & .75 \\ 1\text{ qr.}=\frac{1}{2} & .37\frac{1}{2} \\ \hline \textit{Ans.} & \$7.12\frac{1}{2} \end{array}$$

(9)
$$\begin{array}{r|r} & \$17.25 \\ & 56 \\ \hline & 10350 \\ & 8625 \\ \hline & 966.00 \\ 2\text{ qr.}=\frac{1}{2} & 8.62\frac{1}{2} \\ 1\text{ qr.}=\frac{1}{2} & 4.31\frac{1}{4} \\ \hline \textit{Ans.} & \$978.93\frac{3}{4} \end{array}$$

(10)
$$\begin{array}{r|r} & \$6 \\ & 83 \\ \hline & 498 \\ 2\text{ pk.}=\frac{1}{2} & 3.00 \\ 1\text{ pk.}=\frac{1}{2} & 1.50 \\ 2\text{ qt.}=\frac{1}{4} & .37\frac{1}{2} \\ \hline \textit{Ans.} & \$502.87\frac{1}{2} \end{array}$$

(11)

	$\$4$
	24
	96
2 pk.=$\frac{1}{2}$	2.00
1 pk.=$\frac{1}{2}$	1.00
4 qt.=$\frac{1}{2}$	.50
2 qt.=$\frac{1}{2}$	.25
1 qt.=$\frac{1}{2}$	$.12\frac{1}{2}$
Ans.	$\$99.87\frac{1}{2}$

(12)

	$\$3.20$
	40
	128.00
2 pk.=$\frac{1}{2}$	1.60
1 pk.=$\frac{1}{2}$	.80
4 qt.=$\frac{1}{2}$	.40
2 qt.=$\frac{1}{2}$	.20
1 qt.=$\frac{1}{2}$	.10
1 pt.=$\frac{1}{2}$	.05
Ans.	$\$131.15$

(13)

	$\$2.56$
	17
	1792
	256
	43.52
1 pk.=$\frac{1}{4}$	.64
1 qt.=$\frac{1}{8}$	.08
1 pt.=$\frac{1}{2}$	.04
Ans.	$\$44.28$

(14)

	$\$.24$
	5
	1.20
8 oz.=$\frac{1}{2}$	.12
2 oz.=$\frac{1}{4}$	.03
1 oz.=$\frac{1}{2}$	$.01\frac{1}{2}$
Ans.	$\$1.36\frac{1}{2}$

(15)

	$\$2.56$
	3
	7.68
8 oz.=$\frac{1}{2}$	1.28
4 oz.=$\frac{1}{2}$	.64
1 oz.=$\frac{1}{4}$	.16
8 dr.=$\frac{1}{2}$	.08
4 dr.=$\frac{1}{2}$	.04
2 dr.=$\frac{1}{2}$	.02
1 dr.=$\frac{1}{2}$	.01
Ans.	$\$9.91$

(16)

	$\$3.20$
	17
	2240
	320
	54.40
2 R.=$\frac{1}{2}$	1.60
1 R.=$\frac{1}{2}$	.80
20 P.=$\frac{1}{2}$	.40
10 P.=$\frac{1}{2}$	.20
5 P.=$\frac{1}{2}$	.10
4 P.=$\frac{1}{5}$	.08
Ans.	$\$57.58$

ARTICLE 209.—PERCENTAGE.

7 %=.07 : $4\frac{1}{2}$ %=.045 : $5\frac{1}{4}$ %=.0525 : $10\frac{1}{2}$ %=.105 : $37\frac{1}{2}$ %=.375 : 120 %=1.20

ARTICLE 210.—CASE I.

(2) $7\ \% = .07 : \$175 \times .07 = \$12.25$ *Ans.*
(4) $6\ \% = .06 : \$250 \times .06 = \$15.00$ *Ans.*
(5) $7\ \% = .07 : \$162 \times .07 = \$11.34$ *Ans.*
(6) $5\ \% = .05 : \$118 \times .05 = \$\ 5.90$ *Ans.*
(7) $8\ \% = .08 : \$\ 11 \times .08 = \$\ 0.88$ *Ans.*
(8) $1\ \% = .01 : \$278 \times .01 = \$\ 2.78$ *Ans*
(9) $2\frac{1}{4}\ \% = .02\frac{1}{4} : \$\ 68 \times .02\frac{1}{4} = \$\ 1.53$ *Ans.*

(10) $4\frac{1}{2}\% = .045$: $\$220.50 \times .045 = \$9.922+$ *Ans.*

(11) $7\frac{1}{2}\% = .075$: $\$115.42 \times .075 = \$8.656+$ *Ans.*

(12) $5\frac{3}{4}\% = .05\frac{3}{4}$: $\$243.16 \times .05\frac{3}{4} = \$13.981+$ *Ans.*

(13) $3\frac{1}{4}\% = .03\frac{1}{4}$: $\$1250 \times .03\frac{1}{4} = \$40.625$ *Ans.*

(14) $25\% = .25$: $\$25 \times .25 = \$6.25$ *Ans.*

(15) $101\frac{1}{2}\% = 1.015$: $\$2002 \times 1.015 = \$2032.03$ *Ans.*

(16) $208\% = 2.08$: $\$650 \times 2.08 = \$1352.$ *Ans.*

(17) $1000\% = 10$: $\$24.75 \times 10 = \$247.50$ *Ans.*

(18) $\frac{1}{10}\% = .001$: $\$400 \times .001 = \$0.40$ *Ans.*

(19) $\frac{3}{8}\% = .00\frac{3}{8}$: $\$464 \times .00\frac{3}{8} = \$1.74$ *Ans*

(20) $\frac{1}{12}\% = .00\frac{1}{12}$: $\$1950 \times .00\frac{1}{12} = \$1.62\frac{1}{2}$ *Ans.*

(21) $8\frac{1}{2}\% = .08\frac{1}{2}$: $\$72 \times .08\frac{1}{2} = \$6.12$ · $\$72 - \$6.12$
$= \$65.88$ *Ans.*

(22) $36\% = .36$: $\$800 \times .36 = \$288$: $\$800 - \$288 = \$512$ *Ans.*

(23) $\$56 \times .13\frac{1}{8} = \$7.35$: $\$51 \times .14\frac{2}{3} = \$7.48$: $\$7.48 - 7.35$
$= \$0.13$ *Ans.*

(24) $\$1764 \times .23 = \$405.72$: $\$1764 \times .31 \times \$546.84$: $\$1764 \div .09 = \$158.76$: $\$1764 \times .03\frac{1}{4} = \$57.33$. By adding together the amounts expended, we find their sum is \$1168.65, which, being deducted from \$1764, the remainder is \$595.35, the amount paid for muslin.
Or, by adding the several per cents together, and deducting their sum from 100 % (the whole), the remainder is $33\frac{3}{4}\%$ to be expended in muslin: and $\$1764 \times .33\frac{3}{4} = \$595.35$

(25) $75 \text{ lb.} \times 4 = 300 \text{ lb.}$: $300 \times .12\frac{1}{3} = 37 \text{ lb.}$
$300 \text{ lb.} - 37 \text{ lb.} = 263 \text{ lb.}$, and $14 \text{ cts.} \times 263 = \$36.82$ *Ans.*

(26) $160 \times .35 = 56$: 160 sheep + 56 sheep = 216 sheep *Ans.*

(27) $320 \times .05 = 16$: $320 - 16 = 304$: $304 \times .25 = 76$:
$304 - 76 = 228$ *Ans.*

(28) $18 + 15 + 23 + 12 = 68$: $100 - 68 = 32\%$ left :
$\$800 \times .32 = \$256$ *Ans.*

ARTICLE 211.—CASE II.

(2) $\$15\times.01=\$.15\ :\ \$3.00\div\$.15=20$ *Ans.*

(3) $\$50\times.01=\$.5\ :\ \$6\div\$.5=12$ *Ans.*

(4) $\$75\times.01=\$.75\ :\ \$4.50\div\$.75=6$ *Ans.*

(5) $\frac{3}{9}=\frac{1}{3}=.33\frac{1}{3}=33\frac{1}{3}\ \%$ *Ans.*

(6) $\$25\times.01=\$.25\ :\ \$0.25\div.25=1$ *Ans.*

(7) $\$142.60\times.01=1.426\ :\ \$7.13\div1.426=5$ *Ans.*

(8) $\$9\times.01=\$.09\ :\ \$9\div\$.09=100$ *Ans.*

(9) $\$9\times.01=\$.09\ :\ \$13.50\div\$.09=150$ *Ans.*

(10) $\$243\times.01=\$2.43\ :\ \$8.505\div\$2.43=3.5=3\frac{1}{2}$ *Ans.*

(11) $\$2\times.01=.02\ :\ \$.002\div.02=.1=\frac{1}{10}$ *Ans.*

(12) $\$3532\times.01=\$35.32\ :\ \$13.245\div\$35.32=.375=\frac{3}{8}$ *Ans.*

(13) $\$300\times.01=\$3\ :\ \$25\div\$3=8\frac{1}{3}$ *Ans.*

(14) $\$500\times.01=\$5\ :\ \$75\div\$5=15$ *Ans.*

(15) $\frac{3}{5}=.60=60\ \%$ *Ans.*

(16) 5 bu.=160 qt. $\frac{6}{160}=\frac{3}{80}=.03\frac{3}{4}=3\frac{3}{4}\ \%$ *Ans.*

ARTICLE 213.—COMMISSION.

(1) $\$240\times.05=\$12.00$ *Ans.* **(2)** $\$460\times.02\frac{1}{2}=\$11.50$ *Ans.*

(3) $\$180\times.04=\$7.20\ :\ \$119\times.03=\$3.57\ :$
$\$7.20+\$3.57=\$10.77$ *Ans.*

(4) $\$240\times.03=\$7.20\ :\ \$225\times.05=\$11.25\ :$
$\$7.20+\$11.25=\$18.45$ *Ans.*

(5) $\$480\times.02\frac{1}{4}=\$10.80\ :\ \$275\times.03\frac{1}{5}=\$8.80\ :$
$\$216\times.02\frac{2}{3}=\$\ 5.76\ :\ \$10.80+\$8.80+\$5.76$
$=25.36$ *Ans.*

(6) $\$275\times.03=\$8.25\ :\ \$341\times.15=\$51.15\ :$
$\$964\times.25=\$241.00\ :\ \$217\times.02\frac{1}{4}=4.8825\ :$
$\$8.25+\$51.15+\$241.00+\$4.8825=\$305.2825$ *Ans.*

(7) $\$13\times25=\$325$: $\$325\times.02\frac{1}{2}=\$8.12\frac{1}{2}$ Agent receives : $\$325-\$8.12\frac{1}{2}=\$316.87\frac{1}{2}$ Owner receives.

(8) 1114 lb.$\times$15=16710 lb. : 8 cts.$\times16710=\$1336.80$: $\$1336.80\times.03\frac{1}{4}=\$43.446$ commission : $\$1336.80-\$43.446=\$1293.354$ *Ans.*

(9) $\$15\times250=\$3750$: $\$7\times175=\$1225$: $\$0.25\times1456=\$364.00$: $\$3750+\$1225+\$364=\$5339$: $\$5339\times.03=\$160.17$ com. : $\$5339-\$160.17=\$5178.83$ *Ans.*

Article 214.

(11) $\$1.00+.04=\$1.04$: $\$312\div\$1.04=\$300$ *Ans.*

(12) $\$1.00+.08=\$1.08$: $\$1323.54\div\$1.08=\$1225.50$: $\$1323.54-\$1225.50=\$98.04$ *Ans.*

Article 215.—Insurance.

(1) $\$2250\times.01\frac{1}{2}=\$33.75$: $\$33.75+\$1.00=\$34.75$ *Ans.*

(2) $\$12600+\$14400=\$27000$: $\frac{2}{3}$ of $\$27000=\$18000$: $\$18000\times.04\frac{1}{2}=\$810.00$: $\$810+\$1=\$811$ *Ans.*

(3) $\frac{3}{4}$ of $\$5000=\$3750$: $\$3750\times.00\frac{1}{2}=\$18.75$: $\$18.75+\$1.50=20.25$ *Ans.*

(4) $\frac{4}{7}$ of $21000=\$12000$: $\$12000\times.12\frac{1}{2}=\$1500$: $\$1200\times.01\frac{1}{2}=\$18.00$: $\$1500+\$18+\$1+\$1=\$1520$ *Ans.*

Article 216.

(6) $1-.01=.99$: $\$2475\div.99=\$2500$ *Ans.*

(7) $1-.125=.875$: $\$13125\div.875=\$15000$ *Ans.*

(8) $1-.01\frac{3}{4}=1-.0175=.9825$: $\$2358\div.9825=\$2400$ *Ans.*

(9) $1-.00\frac{3}{8}=1-.00375=.99625$: $\$2287.39\div.99625=\$2296$ · $\$2296-\$2287.39=\$8.61$ *Ans.*
Or, $1-.00\frac{3}{8}=.99\frac{5}{8}=\frac{7.97}{8}$: $\$2287.39\div\frac{7.97}{8}=\$2296$, etc.

ARTICLE 217.—STOCKS.

(1) $\$1400\times.04=\$56$: $\$1400+\$56=\$1456$ *Ans.*
Or, $104\,\%=1.04$: $\$1400\times1.04=\$1456$ *Ans.*

(2) $\$1400\times.96=\$1344.00$ *Ans.*

(3) $\$50\times11=\$550$: $\$550\times.05=\$27.50$: $\$550+\$27.50$
$=\$577.50$ *Ans.*

(4) $\$75\times15=\$1125$: $\$1125\times.10=\$112.50$:
$\$1125-\$112.50=\$1012.50$ *Ans.*

(5) $108\frac{1}{3}-104\frac{1}{2}=3\frac{5}{6}$: $\$1500\times.03\frac{5}{6}=\$57.50$ *Ans.*
Or, $\$1500\times1.04\frac{1}{2}=\$1567.50$: $\$1500\times1.08\frac{1}{3}=\$1625.00$:
$\$1625.00-\$1567.50=\$57.50$ *Ans.*

(6) $3+5\frac{1}{2}=8\frac{1}{2}$: $\$1300\times.08\frac{1}{2}=\$110.50$ *Ans.*
Or, $\$1300\times.97=\$1261.00$ cost : $\$1300\times1.05\frac{1}{2}$
$=\$1371.50$ sold for : $\$1371.50-\$1261.00$
$=\$110.50$ *Ans.*

(7) $4+2\frac{1}{2}=6\frac{1}{2}$: $\$860\times.06\frac{1}{2}=\$55.90$ *Ans.*
Or, $\$860\times1.04=\$894.40$ cost : $\$860\times.97\frac{1}{2}=\$838.50$
sold for : $\$894.40-\$838.50=\$55.90$ *Ans.*

ARTICLE 218.—BROKERAGE.

(1) $\$600\times.01\frac{1}{2}=\$9.00$ *Ans.* **(2)** $\$289\times.01\frac{1}{4}=\$3.61\frac{1}{4}$ *Ans.*

(3) $\$360\times.00\frac{3}{8}=\$1.35$: $\$360-\$1.35=\$358.65$ *Ans.*

(4) $\$134\times.02\frac{1}{2}=\$3.35$: $\$134-\$3.35=\$130.65$ *Ans.*

(5) $\$200\times.00\frac{5}{8}=\$1.25$: $\$200+\$1.25=\$201.25$ *Ans.*

(6) $\frac{1}{2}+\frac{1}{4}=\frac{3}{4}$. $\$500\times.00\frac{3}{4}=\$3.75$ *Ans.*
Or, $\$500\times.00\frac{1}{2}=\$2.50$ gain in buying. $\$500\times.00\frac{1}{4}$
$=\$1.25$ gain in selling. $\$2.50+\$1.25=\$3.75$ *Ans.*

ARTICLE 220.—CASE I.

(3) $\$65\times.05=\$3.25$: $\$3.25\times4=\$13.00$ *Ans.*

(4) $\$200\times.08=\$16.00$ *Ans.* **(5)** $\$150\times.05=\$7.50$ *Ans.*

(6) $\$300\times.06=\$18.00 : \$18.00\times2=\$36.00$ *Ans.*

(7) $\$275\times.06=\$16.50 : \$16.50\times3=\$49.50$ *Ans.*

(8) $\$187.50\times.05=\$9.375 : \$9.375\times4=\$37.50$ *Ans.*

(9) $\$233.80\times.06=\$14.028 : \$14.028\times10=\$140.28$ *Ans.*

(11) $\$215.00\times.06=\$12.90 : \$215+\$12.90=\$227.90$ *Ans.*

(12) $\$45.00\times.08=\$3.60 : \$3.60\times2=\$7.20 :$
$\$45+\$7.20=\$52.20$ *Ans.*

(13) $\$80.00\times.07=\$5.60 : \$5.60\times4=\$22.40 :$
$\$80+\$22.40=\$102.40$ *Ans.*

(14) $\$420.00\times.05\frac{1}{3}=\$22.40 : \$420.00+\$22.40=\$442.40$ *Ans.*

(15) $\$237.16\times.03\frac{3}{4}=\$8.8935 : \$8.8935\times2=\$17.787 :$
$\$237.16+\$17.787=\$254.947$ *Ans.*

(16) $\$74.75\times.04=\$2.99 : \$2.99\times5=\$14.95 :$
$\$74.75+\$14.95=\$89.70$ *Ans.*

(17) $\$85.45\times.06=\$5.127 : \$5.127\times4=\$20.508 :$
$\$85.45+\$20.508=\$105.958$ *Ans.*

(18) $\$325.00\times.05\frac{2}{5}=\$17.55 : \$17.55\times3=\$52.65 :$
$\$325+\$52.65=\$377.65$ *Ans.*

(19) $\$129.36\times.04\frac{3}{8}=\$5.6595 : \$5.6595\times4=\$22.638 :$
$\$129.36+\$22.638=\$151.998$ *Ans.*

ARTICLE 221.—CASE II.

(2) $\$240\times.08=\$19.20 :$ 2 mon.$=\frac{2}{12}=\frac{1}{6}$ yr. :
$\frac{1}{6}$ of $\$19.20=\$3.20$ *Ans.*

(4) $\$360\times.06=\$21.60 :$ $\frac{1}{12}$ of $\$21.60=\$1.80 :$
2 da.$=\frac{2}{30}=\frac{1}{15}$ mon. : $\frac{1}{15}$ of $\$1.80=\$.12$ *Ans.*

(6) $\$86\times.06=\$5.16 :$ 3 mon.$=\frac{1}{4}$ yr. : $\frac{1}{4}$ of $\$5.16=\$1.29$ *Ans.*

(7) $\$50\times.08=\$4.00 :$ 4 mon.$=\frac{1}{3}$ yr. : $\frac{1}{3}$ of $\$4.00=\$1.33\frac{1}{3}$ *Ans.*

(8) $\$150.25\times.08=\$12.02 :$ 6 mon.$=\frac{1}{2}$ yr. :
$\frac{1}{2}$ of $\$12.02=\$6.01$ *Ans.*

(9) $\$360\times.05=\$18.00 :$ 7 mon.$=\frac{7}{12}$ yr. :
$\frac{7}{12}$ of $\$18.00=\$10.50$ *Ans.*

(10) $\$204\times.07=\$14.28$: 11 mon.$=\frac{11}{12}$ yr. : $\frac{11}{12}$ of $\$14.28=\$13.09$ *Ans.*

(11) $\$726\times.06=\$43.56$: $\frac{1}{12}$ of $\$43.56=\$3.63$: 10 da.$=\frac{1}{3}$ mon. : $\frac{1}{3}$ of $\$3.63=\$1.21$ *Ans.*

(12) $\$1200\times.06=\$72.00$: $\frac{1}{12}$ of $\$72.00=\$6.00$: 15 da.$=\frac{1}{2}$ mon. : $\frac{1}{2}$ of $\$6.00=\$3.00$ *Ans.*

(13) $\$180\times.08=\$14.40$: $\frac{1}{12}$ of $\$14.40=\$1.20$: 19 da.$=\frac{19}{30}$ mon. : $\frac{19}{30}$ of $\$1.20=\$0.76$ *Ans.*

(14) $\$240\times.07=\$16.80$: $\frac{1}{12}$ of $\$16.80=\$1.40$: 27 da.$=\frac{27}{30}=\frac{9}{10}$ mon. : $\frac{9}{10}$ of $\$1.40=\$1.26$ *Ans.*

(15) $\$100.80\times.05=\$5.04$: $\frac{1}{12}$ of $\$5.04=\$.42$: 28 da.$=\frac{28}{30}=\frac{14}{15}$ mon. : $\frac{14}{15}$ of $\$0.42=\$0.392$ *Ans.*

(16) $\$228\times.06=\$13.68$: 9 mon.$=\frac{9}{12}=\frac{3}{4}$ yr. : $\frac{3}{4}$ of $\$13.68=\$10.26$: $\$228+\$10.26=\$238.26$ *Ans.*

(17) $\$137.50\times.06=\$8.25$: 8 mon.$=\frac{8}{12}=\frac{2}{3}$ yr. : $\frac{2}{3}$ of $\$8.25=\$5.50$: $\$137.50+\$5.50=\$143.00$ *Ans.*

(18) $\$150\times.05=\$7.50$: $\frac{1}{12}$ of $\$7.50=\$.625$: 18 da.$=\frac{18}{30}=\frac{3}{5}$ mon. : $\frac{3}{5}$ of $\$.625=\$.375$: $\$150+\$.375=\$150.375$ *Ans.*

(19) $\$360\times.06=\$21.60$: $\frac{1}{12}$ of $\$21.60=\$1.80$: 11 da.$=\frac{11}{30}$ mon. : $\frac{11}{30}$ of $\$1.80=\$.66$ $\$360+\$.66=\$360.66$ *Ans.*

(20) $\$264\times.06=\$15.84$: $\frac{1}{12}$ of $\$15.84=\$1.32$: 9 da.$=\frac{9}{30}=\frac{3}{10}$ mon. : $\frac{3}{10}$ of $\$1.32=\$.396$: $\$264+\$.396=\$264.396$ *Ans.*

ARTICLE 222.

(2)
$$\begin{array}{r|r} & \$350\\ & .04\\ \hline & 14.00\\ & 7\\ \hline & 98.00\\ 3\text{ mon.}=\frac{1}{4} & 3.50\\ \hline \end{array}$$
Ans. \$101.50

(3)
$$\begin{array}{r|r} & \$150\\ & .06\\ \hline & 9.00\\ & 4\\ \hline & 36.00\\ 2\text{ mon.}=\frac{1}{6} & 1.50\\ \hline \end{array}$$
Ans. \$37.50

(4)
$$\begin{array}{r|r} & 375.40\\ & .06\\ \hline & 22.5240\\ 6\text{ mon.}=\frac{1}{2} & 11.262\\ 2\text{ mon.}=\frac{1}{3} & 3.754\\ \hline \end{array}$$
Ans. \$37.540

(5) \$92.75
.06
5.5650
3
16.695
4 mon.=$\frac{1}{3}$ 1.855
1 mon.=$\frac{1}{4}$ .463+
Ans. \$19.013+

(6) \$500
.06
30.00
1 mon.=$\frac{1}{12}$ 2.50
15 da. = $\frac{1}{2}$ 1.25
3 da. = $\frac{1}{5}$ .25
Ans. \$34.00

(7) \$560
.08
44.80
2
89.60
4 mon.=$\frac{1}{3}$ $14.93\frac{1}{3}$
15 da.= $\frac{1}{8}$ $1.86\frac{2}{3}$
Ans. \$106.40

(8) \$750
.06
\$45.00
4
180.00
3 mon.= $\frac{1}{4}$ 11.25
6 da. =$\frac{1}{15}$ 0.75
Ans. \$192.00

(9) \$456
.05
\$22.80
3
68.40
4 mon.=$\frac{1}{3}$ 7.60
1 mon.=$\frac{1}{4}$ 1.90
15 da. =$\frac{1}{2}$ .95
3 da. =$\frac{1}{5}$ .19
Ans. \$79.04

(10) \$216
.10
21.60
5
108.00
6 mon.=$\frac{1}{2}$ 10.80
1 mon.=$\frac{1}{6}$ 1.80
15 da. =$\frac{1}{2}$ .90
10 da. =$\frac{1}{3}$ .60
2 da. =$\frac{1}{5}$ .12
Ans. \$122.22

(11) \$380
.15
1900
380
\$57.00
3
171.00
6 mon.= $\frac{1}{2}$ 28.50
3 mon.= $\frac{1}{2}$ 14.25
9 da. =$\frac{1}{10}$ 1.425
Ans. \$215.175

(12) \$300
.06
\$18.00
3
54.00
6 mon.=$\frac{1}{2}$ 9.00
2 mon.=$\frac{1}{3}$ 3.00
Int., \$66.00
300.00
Ans. \$366.00

(13) \$250
.06
15.00
6 mon.=$\frac{1}{2}$ 7.50
1 mon.=$\frac{1}{6}$ 1.25
Int., \$23.75
250.00
Ans. \$273.75

(14)

$205.25
.06
$12.3150
2
24.630
6 mon. = $\frac{1}{2}$ 6.1575
2 mon. = $\frac{1}{3}$ 2.0525
15 da. = $\frac{1}{4}$ .5131
Int., $33.3531+
205.25
Ans. $238.6031+

(15)

$150.62
.05
$7.5310
3
22.593
4 mon. = $\frac{1}{3}$ 2.510+
1 mon. = $\frac{1}{4}$ .627
12 da. = $\frac{1}{10}$ .251
Int., $25.981+
150.62
Ans. $176.601+

(16)

$210.25
.07
$14.7175
2
29.4350
6 mon. = $\frac{1}{2}$ 7.3587+
1 mon. = $\frac{1}{6}$ 1.2264
20 da. = $\frac{1}{9}$ .8176
Int., $38.8377+
210.25
Ans. $249.0877+

(17)

$57.85
.05
$2.8925
2
5.7850
3 mon. = $\frac{1}{4}$ .7231+
15 da. = $\frac{1}{6}$ .1205
6 da. = $\frac{1}{15}$ .0482
2 da. = $\frac{1}{3}$ .0160
Int., $6.6928+
57.85
Ans. $64.5428+

(18)

yr.	mon.	da.
1849	4	19
1847	1	9
2	3	10

$150
.06
9.00
2
18.00
3 mon. = $\frac{1}{4}$ 2.25
10 da. = $\frac{1}{9}$ .25
Ans. $20.50

(19)

yr.	mon.	da.
1849	4	27
1848	2	15
1	2	12

$240
.08
19.20
2 mon. = $\frac{1}{6}$ 3.20
12 da. = $\frac{1}{5}$ .64
Ans. $23.04

(20)

yr. mon. da.
1845 8 28
1843 5 14
2 3 14

$180
.07
12.60
2
25.20
3 mon.=$\frac{1}{4}$ | 3.15
10 da. =$\frac{1}{9}$ | .35
2 da. =$\frac{1}{5}$ | .07
2 da. =$\frac{1}{5}$ | .07
Ans. $28.84

(21

mon. da.
11 27
7 3
4 24

$137.50
.09
$12.3750
4 mon.=$\frac{1}{3}$ | 4.125
20 da. =$\frac{1}{6}$ | .6875
4 da. =$\frac{1}{5}$ | .1375
Ans. $4.9500

(22)

mon. da.
8 28
3 1
5 27

$125.40
.08½
100320
6270
$10.6590
4 mon.=$\frac{1}{3}$ | 3.553
1 mon.=$\frac{1}{4}$ | .888+
15 da. =$\frac{1}{2}$ | .444
10 da. =$\frac{1}{3}$ | .296
2 da. =$\frac{1}{5}$ | .059
Int., $5.240+
125.40
Ans. $130.64+

(23)

yr. mon. da.
1848 3 9
1847 8 2
7 7

$234.60
.05¼
11.7300
5865
12.3165
6 mon.=$\frac{1}{2}$ | 6.15825
1 mon.=$\frac{1}{6}$ | 1.02637+
6 da. =$\frac{1}{5}$ | .20527
1 da. =$\frac{1}{6}$ | .03421
Int., $7.42410
234.60
Ans. $242.024+

(24)

yr. mon. da.
1847 7 24
1846 10 25
8 29

$153.80
.05
7.6900
6 mon.=$\frac{1}{2}$ | 3.845
2 mon.=$\frac{1}{3}$ | 1.2816+
20 da. =$\frac{1}{3}$ | .4272
5 da. =$\frac{1}{4}$ | .1068
4 da. =$\frac{1}{5}$ | .0854
Int., $5.7460
153.80
Ans. $159.546+

ARTICLE 223.

(3) 16 mon.
Ans. $.08

13 mon.
Ans. $.065

4 mon. 18 da.
Ans. $.023

7 mon. 12 da.
$.035
2
Ans. $.037

10 mon. 13 da.
Ans. $\$.052\frac{1}{6}$

5 mon. 17 da.
$.025
$2\frac{5}{6}$
Ans. $\$.027\frac{5}{6}$

11 mon.
Ans. $.055

2 mon. 1 da.
Ans. $\$.010\frac{1}{6}$

9 mon. 3 da.
Ans. $\$.045\frac{1}{2}$

14 mon. 4 da.
Ans. $\$.070\frac{2}{3}$

17 mon. 27 da.
$.085
$4\frac{1}{2}$
Ans. $\$.089\frac{1}{2}$

33 mon. 20 da.
$.165
$3\frac{1}{3}$
Ans. $\$.168\frac{1}{3}$

ARTICLE 224.

(2) $.04×65=$2.60 *Ans.*
(3) $.055×36=$1.98 *Ans.*
(4) $.08×28=$2.24 *Ans.*
(5) $.125×500=$62.50 *Ans.*
(6) $.022×75=$1.65 *Ans.*
(7) $\$.048\frac{5}{6}\times 186=\$9.083$ *Ans.*
(8) $.072×125=$9.00 *Ans.*
(9) $.204×210.25=$42.891 *Ans.*
(10) $\$.087\frac{1}{2}\times 134.45=\$11.764+$ *Ans.*
(11) $\$.138\frac{1}{6}\times 144.24=\$19.929+$ *Ans.*

ARTICLE 226.

(2) Principal	$200.00
Interest to 1st payment, (12 mon.,)	12.00
Amount due, Jan. 1st, 1846	212.00
1st payment, (greater than int.,) to be deducted	70.00
Balance due Jan. 1st, 1846	142.00
Interest for 1 year	8.52
Ans. Amount due Jan. 1st, 1847 .	$150.52

(3) Principal	$300.00
Interest to 1st payment, (6 mon.,)	9.00
Amount due Jan. 1st, 1844	309.00
1st payment, (greater than int.,) to be deducted	109.00
Balance due Jan. 1st, 1844,	200.00
Interest to 2d payment, July 1st, 1844 (6 mon.,)	6.00
	$206.00
2d payment, (greater than int.,) to be deducted	100.00
Balance due July 1st, 1844	106.00
Interest to Jan. 1st, 1845, (6 mon.,)	3.18
Ans. Amount due Jan. 1st, 1845	$109.18

(4) Principal		$150.00
Interest to 1st payment (1 yr. 4 mon.,)		12.00
Amount due Sept. 10th, 1851		162.00
1st payment (greater than int.,) to be deducted		32.00
Balance due Sept. 10th, 1851		130.00
Sept. 10th, 1852, paid a sum less than the interest then due	$6.80	
Interest from Sept. 10th, 1851, to Nov. 10th, 1852, (1 yr. 2 mon.,)		9.10
		139.10
Deduct 2d payment		6.80
Ans. Amount due Nov. 10th, 1852		$132.30

(5) Principal		$200.00
June 5th, 1842, paid a sum less than the interest then due	$20.00	
Dec. 5th, 1842, paid, (making in all a sum greater than the interest,)	50.50	
Interest from Mar. 5th, 1841, to Dec. 5th, 1842, (1 yr. 9 mon.,)		35.00
		$235.00
Deduct the sum of the 1st and 2d payments		70.50
Balance due Dec. 5th, 1842,		164.50
Interest on balance to settlement (1 yr. 6 mon.,)		24.675
Ans. Amount due June 5th, 1844		$189.175

(6) Principal		$250.00
June 1st, 1845, paid a sum less than int., then due	$6.00	
Jan. 1st, 1846, paid (making in all a sum greater than int. then due)	21.50	
Interest to 2d payment, (12 mon.,)		17.50
		267.50
Deduct the sum of the 1st and 2d payments		27.50
Balance due Jan. 1st, 1846		240.00
Interest to July 1st, 1846, (6 mon.,)		8.40
Ans. Amount due July 1st, 1846		$248.40

(7) Principal		$180.00
Interest to 1st payment, (6 mon.)		5.40
Amount due Feb. 1st, 1845		185.40
1st payment, (greater than interest,)		25.40
Balance due Feb. 1st, 1845		160.00
2d payment, less than interest then due	$4.30	
3d payment	30.00	
Int. on bal. to Jan. 1st, 1846, (11 mon.,)		8.80
		168.80
Deduct sum of 2d and 3d payments		34.30
Balance due Jan. 1st, 1846		$134.50
Interest to July 1st, 1846, (6 mon.,)		4.035
Ans. Amount due July 1st, 1846		$138.535

(8) Principal		$400.00
Sept. 1st, 1845. Paid, a sum less than interest then due	$10.00	
Jan. 1st, 1846. Paid in all, greater than interest then due	30.00	
Interest to Jan. 1st, 1846, (10 mon.,)		20.00
		$420.00
Deduct sum of 1st and 2d payments		40.00
Balance due Jan. 1st, 1846		$380.00
July 1st, 1846. Paid a sum less than interest then due	$11.00	
Sept. 1st, 1846. Paid in all, greater than interest then due	80.00	
Interest to Sept. 1st, 1846, (8 mon.,)		15.20
		$395.20
Deduct sum of 3d and 4th payments		91.00
		304.20
Interest to March 1st, 1847, (6 mon.,)		9.126
Ans. Amount due March 1st, 1847		$313.326

(9) Principal		$450.00
1st payment, Jan. 1st, 1847, less than interest then due	$20.00	
2d payment, April 1st, 1847, less than interest then due	14.00	
3d payment, July 16th, 1847, in all, more than interest then due	31.00	
Interest to July 16th, 1847, (15 mon.,)		45.00
		495.00
Deduct sum of 1st three payments		65.00
Balance due July 16th, 1847		$430.00
4th payment, Dec. 25th, 1847, less than interest then due	$10.00	
5th payment, July 4th, 1848, less than interest then due	18.00	
Interest to June 1st, 1849, (22 mon. 15 da.,)		64.50
		494.50
Deduct sum of 4th and 5th payments		28.00
Ans. Balance due June 1st, 1849		$466.50

(10) Principal		$1000.00
1st payment, May 1st, 1840, less than interest then due	$18.00	
2d payment, Sept. 4th, 1840, less in all than interest then due	20.00	
3d payment, Dec. 16th, 1840, less in all than interest then due	15.00	
4th payment, Ap. 10th, 1841, less in all than interest then due	21.00	
5th payment, July 13th, 1841, more in all than interest then due	118.00	
Interest to July 13th, 1841, (18 mon. 12 da.,)		92.00
		$1092.00
Deduct sum of 1st five payments		192.00
Balance due July 13th, 1841,		900.00
Interest to Dec. 23, 1841, (5 mon. 10 da.,)		24.00
Amount due Dec. 23, 1841		924.00
6th payment, (greater than int.,)		324.00
Balance due Dec. 23, 1841,		600.00
Interest to Oct. 1st, 1843, (1 yr. 9 mon. 8 da.,)		63.80
Ans. Amount due Oct. 1st, 1843		$663.80

Remark.—In accordance with suggestion preceding Example 2d, in Arith., in all the preceding solutions the computation of interest has been deferred, until the period when a payment, or the sum of two or more payments, exceeds the interest due.

ARTICLE 227.

(1) Principal		$320.00
Interest to Jan. 1st, 1847, (1 yr.)		19.20
		339.20
1st payment, May 1st, 1846	$50.00	
Interest to Jan. 1st, 1847, (8 mon.,)	2.00	
2d payment, Nov. 16th, 1846	100.00	
Int. to Jan. 1st, 1847, (1 mon. 15 da.,)	.75	
	$152.75	152.75
Ans. Balance due Jan. 1st, 1847		$186.45

(2) Principal		$540.00
Interest to Jan. 1st, 1848, (10 mon.,)		36.00
		$576.00
1st payment, May 1st, 1847	$90.00	
Int. to Jan. 1st, 1848, (8 mon.,)	4.80	
2d payment, July 1st, 1847	100.00	
Int. to Jan. 1st, 1848, (6 mon.,)	4.00	
3d payment, Aug. 1st, 1847	150.00	
Int. to Jan. 1st, 1848, (5 mon.,)	5.00	
4th payment, Oct. 11th, 1847	180.00	
Int. to Jan. 1st, 1848, (2 mon. 20 da.,)	3.20	
	$537.00	537.00
Ans. Balance due Jan. 1st, 1848		$39.00

ARTICLE 228.

(1) Principal of the note	$875.000
Interest to Aug. 10, 1834, (43 mon.)	188.125
Amount	$1063.125
1st payment deduct	260.000
Due Aug. 10, 1834, forming a new principal,	$803.125
Interest to Dec. 16, 1835, (16 mon. 6 da.,)	65.053
	$868.178
2d payment deduct	300.000
Due Dec. 16, 1835, forming a new principal,	$568.178
Interest to Dec. 16, 1836, the time between the payments being less than 1 year	34.090
	$602.268
Amount of $50 from March 1, 1836, to Dec. 16, 1836, (9 mon. 15 da.,) to be deducted	52.375
Due Dec. 16, 1836, forming a new principal,	$549.893
Interest from Dec. 16, 1836, to Dec. 16, 1837	32.993
	$582.886
Amount of $150 from July 1, 1837, to Dec. 16, 1837, (5 mon. 15 da.,)	154.125
Due Dec. 16, 1837, forming a new principal,	$428.761
Interest to Sept. 1, 1838, (8 mon. 15 da.,)	18.222
Ans. Due on settlement	$446.983+

ARTICLE 230.

(2) \$ 60×.05=\$ 3.00 : \$12÷\$ 3=4 yr. *Ans.*

(3) \$375×.08=\$30.00 : \$90÷\$30=3 yr. *Ans.*

(4) \$225×.04=\$ 9.00 : \$66÷\$ 9=$7\frac{1}{3}$ yr.
$7\frac{1}{3}$ yr.=7 yr. 4 mon. *Ans.*

(5) \$250×.06=\$15.00 : \$34.50÷\$15
=2.3 yr.=2 yr. 3 mon. 18 da. *Ans.*

(6) \$60×.06=\$3.60 : \$13.77÷\$3.6
=3.825 yr.=3 yr. 9 mon. 27 da. *Ans.*

(7) \$800—\$500=\$300 : \$500×.10=\$50 :
\$300÷\$50=6 yr. *Ans.*

(8) \$798—\$600=\$198 : \$600×.09=\$54 :
\$198÷\$54=$3\frac{2}{3}$ yr.=3 yr. 8 mon. *Ans.*

(9) \$200×.06=\$12 : \$200÷\$12=$16\frac{2}{3}$ yr.=16 yr. 8 mon. *Ans.*

(10) \$100÷\$4=25 yr. *Ans.* **(11)** \$100÷\$5=20 yr. *Ans.*

(12) \$100÷\$7=$14\frac{2}{7}$ yr. *Ans.* **(13)** \$100÷\$8=$12\frac{1}{2}$ yr. *Ans.*

(14) \$100÷\$10=10 yr. *Ans.* **(15)** \$100÷\$12=$8\frac{1}{3}$ yr. *Ans.*

(16) \$100÷\$16=$6\frac{1}{4}$ yr. *Ans.*

(17) \$100×3=\$300 : \$300—\$100=\$200 : \$200÷\$15
=$13\frac{1}{3}$ yr. *Ans.*
\$100×4=\$400 : \$400—\$100=\$300 : \$300÷\$15
=20 yr. *Ans.*

ARTICLE 231.

(2) \$80×.01=\$.80 : \$.80×5=\$4.00 : \$28÷\$4=7 % *Ans.*

(3) \$300×.01=\$3 : \$3×$1\frac{2}{3}$=\$5 : \$30÷\$5=6 % *Ans.*

(4) \$1000×.01=10 : \$10×$2\frac{1}{2}$=\$25 : \$200÷\$25=8 % *Ans.*

(5) \$260×.01=\$2.60 : \$2.60×2=5.20 :
\$23.40÷\$5.2=4.5=$4\frac{1}{2}$ % *Ans.*

(6) \$640×.01=\$6.40 : \$6.40×6=\$38.40 :
\$110.40÷\$38.40=2.875=$2\frac{7}{8}$ % *Ans.*

ARTICLE 232.

(2) $\$.05\times3=\$.15 : \$8.25\div\$.15=\$55$ *Ans.*

(3) $\$.05\times3=\$.15 : \$341.25\div\$.15=\$2275$ *Ans.*

(4) Int. of $\$1.00$ for 16 mon., at 6 % (Art. 223) $=\$.08$:
$\$2.26\div\$.08=\$28.25$ *Ans.*

(5) $\$1023.75\div\$.05=\$20475$ *Ans.*

(6) Int. of $\$1$ for 12 yr. 3 mon. 20 da. at 9 % (Art. 222)
$=\$1.1075 : \$525.398\div\$1.1075\doteq\$474.40$ *Ans.*

ARTICLE 233.

(2) $\$200\times.08=16 : \$200+\$16=\$216$, 1st yr.
$\$216\times.08=\$17.28 : \$216+\$17.28=\$233.28$, 2d yr.
$\$233.28-\$200=\$33.28$ *Ans.*

(3) $\$500\times.06=\$30 : \$500+\$30=\$530$, 1st yr.
$\$530\times.06=\$31.80 : \$530+\$31.80=\$561.80$, 2d yr.
$\$561.80\times.06=\$33.708$:
$\$561.80+\$33.708=\$595.508$ *Ans.*

(4) $\$800\times.06=\$48.00 : \$48+\$800=\$848$, 1st yr.
$\$848\times.06=\$50.88 : \$50.88+\$848=\$898.88$, 2d yr.
$\$898.88\times.06=\$53.9328 : \$53.9328+\$898.88$
$=\$952.8128$, 3d yr.
$\$952.8128\times.06=\$57.168+ : \$57.168+952.8128$
$=1009.98+$ *Ans.*

(5) $\$250\times.06=\$15 : \$15+\$250=\$265$, 1st yr.
$\$265\times.06=\$15.90 : \$15.90+\$265=\$280.90$, 2d yr.
$\$280.90\times.06=\$16.854 : \$16.854+\$280.90$
$=\$297.754$, 3d yr.
$\$297.754-\$250=\$47.754$ *Ans.*

(6) $\$300\times.05=\$15 : \$15+\$300=\$315$, 1st yr.
$\$315\times.05=\$15.75 : \$15.75+\$315=\$330.75$, 2d yr.
$\$330.75\times.05=\$16.5375 : \$16.5375+\$330.75$
$=\$347.2875$, 3d yr.
$\$347.2875\times.05=\$17.364+ : \$17.364+\$347.287$
$=\$364.65+$, 4th yr.
$\$364.65-\$300=64.65+$ *Ans.*

(7) Int. of $1.00 for 6 mon. at 6 % (Art. 233) =$.03
$.03×200=$6 : $6+$200=$206, 1st 6 mon.
$.03×206=$6.18 : $6.18+$206=$212.18, 2d 6 mon.
$.03×212.18=$6.3654 : $6.3654+$212.18=$218.5454,
3d 6 mon.
$.03×218.5454=$6.5563+ : $6.5563+$218.5454
=$225.101+, 4th 6 mon.
$225.101—$200=$25.101 *Ans.*

(8) 20 % per annum is equal to 5 % quarterly, the question then is the same as to find the amount of $500 for 8 years, at 5 % per annum.
$500×.05=$25 : $25+$500=$525, 1st quarter.
$525×.05=$26.25 : $26.25+$525=$551.25, 2d quar.
$551.25×.05=$27.5625 : $27.5625+$551.25
=$578.8125, 3d qr.
$578.8125×.05=$28.9406+ : $28.9406+$578.8125
=$607.7531, 4th qr.
$607.7531×.05=$30.3876+ : $30.3876+$607.7531
=$638.1407, 5th qr.
$638.1407×.05=$31.907 + : $31.907+$638.1407
=$670.0477, 6th qr.
$670.0477×.05=$33.5023+ : $33.5023+$670.0477
=$703.55, 7th qr.
$703.55×.05=35.177+ : $35.177+$703.55
=$738.727+ *Ans.*

(9) $300×.06=$18.00 : $18.00+$300=$318, 1st yr.
$318×.06=$19.08 : $19.08+$318=$337.08, 2d yr.
$337.08×.06=$20.2248 : $20.2248÷2=$10.1124 :
$10.1124+$337.08=$347.1924 : $347.1924—$300
=$47.1924 *Ans.*

(10) $1000×.06=$60 : $60+$1000=$1060, 1st yr.
$1060×.06=$63.60 : $63.60+$1060=$1123.60, 2d yr.
Int. of $1123.60 for 8 mon. 15 da. (Art. 232) =$47.753 :
$47.753+$1123.60=$1171.353 : $1171.353—$1000
=$171.353 *Ans.*

ARTICLE 234.

(12) Tab. num. 1.790848×345=$617.84+ *Ans.*

(14) 30 yr.=20 yr.+10 yr. Tab. num. for 20 yr.=3.207135 :
Tab. num. for 10 yr.=1.790848. 3.207135×890
=2854.35015 : 2854.35015×1.790848=$5111.70+
$5111.70—$890=$4221.70+ *Ans.*

(15) 70 yr.=25 yr.+25 yr.+20 yr. Tab. num. for 25 yr.
=3.386355 : Tab. num. for 20 yr.=2.653298.
3.386355×200=677.271 : 3.386355×677.271
=2293.48 : 2.653298×2293.48
=$6085.28+ : $6085.28—$200.=$5885.28+ *Ans.*

ARTICLE 236.—DISCOUNT.

(2) $.04×2=$.08 : $1+$.08=$1.08 : $81÷$1.08
=$75 *Ans.*

(3) $.06×5=$.30 : $1+$.30=$1.30 : $1300÷$1.30
=$1000 *Ans.*

(4) $.06×3=$.18 : $1+$.18=$1.18 : $4720÷$1.18
=$4000 *Ans.*

(5) $1+$.06=$1.06 : $257.50÷$1.06=$242.924+ *Ans.*

(6) Int. of $1 for 1 yr. 10 mon. (Art. 223)=$.11 :
$1+$.11=$1.11 : $199.80÷$1.11=$180.00 *Ans.*

(7) Int. of $1 for 5 yr. 10 mon. (Art. 223) =$.35 :
$1+$.35=$1.35 : $6.75÷$1.35=$500 *Ans.*

(8) Int. of $1 for 5 mon. (Art. 223) =$.025 :
$1+$.025=$1.025 : $307.50÷$1.025=$300.00 *Ans.*

(9) Int. of $1 for 7 yr. 9 mon. 20 da. (Art. 223) =$.468⅓ :
$1+$.468⅓=$1.468⅓ : $493.20÷$1.468⅓
=$335.89+ *Ans.*

(10) $.06×4=$.24 : $1+$.24=$1.24 : $496÷$1.24
=$400, present worth. $496—$400=$96 *Ans.*

(11) $.06×2=$.12 : $1+$.12=$1.12 : $276.64÷$1.12=$247,
present worth. $276.64—$247.00=$29.64 *Ans.*

(12) Int. of $1 for 3 yr. 4 mon. (Art. 223)=$.20 :
$1+$.20=$1.20 : $330÷$1.20=$275 present worth.
$330—$275=$55 *Ans.*

(13) Int. of $1 for 8 mon. (Art. 223)=$.04 :
$1+$.04=$1.04 : $260÷$1.04=$250 *Ans.*

(14) $1+$.12=$1.12 : $235.20÷$1.12=$210 *Ans.*

(15) $1+$.07=$1.07 : $401.25÷$1.07=$375 present worth.
$401.25—$375=$26.25 *Ans.*

(16) $1080×.06=$64.80 : $64.80×10=$648=interest.
$.06×10=$.60 : $1+$.60=$1.60 : $1080÷$1.60
=$675. $1080—$675=$405 discount. $648—$405
=$243 *Ans.*

(17) 10/12 of $.12=$.10 : $1+$.10=$1.10 : $1221÷$1.10
=$1110. $1122—$1110=$12 *Ans.*

ARTICLE 237.

(18) $956.34÷3=$318.78, each payment.
$.05+$1=$1.05 : $318.78÷$1.05=$303.60, present worth 1st payment.
$.05×2=$.10 : $1+$.10=$1.10 : $318.78÷$1.10
=$289.80, present worth 2d payment.
$.05×3=$.15 : $1+$.15=$1.15 : $318.78÷$1.15
=$277.20, present worth 3d payment.
$303.60+$289.80+$277.20=$870.60 *Ans.*

(19) $1440÷2=$720, 1st payment : $1440÷3=$480, 2d payment : $1440—($720+$480)=$240, last payment.
Int. of $1 for 3, 6, and 9 mon. (Art. 223) is
$.015, $.03, $.045
$1+$.015=$1.015 : $720÷$1.015=$709.3596+,
present worth 1st payment.
$1+$.03=$1.03 : $480÷$1.03=$466.0194+,
present worth 2d payment.
$1+$.045=$1.045 : $240÷$1.045=$229.665+,
present worth 3d payment.
Adding the present worth of the three payments together, their sum is $1405.044+ *Ans.*

(20) $700—($60+$180+$260)=$200, last payment.
Int. of $1 for 6, 12, 18, and 24 mon. (Art. 223) is
$.03, $.06, $.09, $.12
$1+.03=$1.03 : $60÷$1.03=$58.2524+, present worth 1st payment.
$1+$.06=$1.06 : $180÷$1.06=$169.8113+, present worth 2d payment.
$1+$.09=$1.09 : $260÷$1.09=$238.5321+, present worth 3d payment.
$1+$.12=$1.12 : $200÷$1.12=$178.5714+, present worth 4th payment.
Sum of payments $645.167+ *Ans.*

ARTICLE 238.

(1) $.06×5=$.30 : $1+$.30=$1.30 : $650÷$1.30 =$500 *Ans.*

(2) $.05×9=$.45 : $1+$.45=$1.45 : $725÷$1.45 =$500 *Ans.*

(3) $.05×4=$.20 : $1+$.20=$1.20 : $571.20÷$1.20 =$476, principal.
$571.20—$476=$95.20, interest. *Ans.*

(4) $.06×2½=$.15 : $1+$.15=$1.15 : $690÷$1.15 =$600, principal.
$690—$600=$90, interest. *Ans.*

ARTICLE 240.—BANK DISCOUNT.

(2) $137× 93÷60=212+. Int.=$2.12+*Ans.*
(3) $1780×93÷60=89×31=2759. Int.=$27.59 *Ans.*
(4) $375× 33÷60=75×11÷4=206¼. Int.=$2.06¼ *Ans.*
(5) $165× 63÷60=33×21÷4=173¼. Int.=$1.73¼ *Ans.*
(6) $140×123÷60= 7×41=287. Int.=$2.87 *Ans.*
(7) $ 80×183÷60= 4×61=244. Int.=$2.44 *Ans.*
(8) $180× 33÷60= 3×33=99. Int.=99 cts.
$180—$.99=$179.01 *Ans.*

(9) $\$960\times33\div60=16\times33=528$. Int.$=\$5.28$
$\$960-\$5.28=\$954.72$ *Ans.*

(10) $\$875\times93\div60=175\times31\div4=1356\frac{1}{4}$. Int.$=\$13.56\frac{1}{4}$
$\$875-\$13.56\frac{1}{4}=\$861.43\frac{3}{4}$ *Ans.*

(11) $\$3900\times63\div60=65\times63=4095$. Int.$=\$40.95$
$\$3900-\$40.95=\$3859.05$ *Ans.*

(12) $\$2580\times103\div60=43\times103=4429$. Int. at 6 %.
$=\$44.29$ Int. at 5 %. (Art. 225, Rem. 3.,)
$=\frac{5}{6}$ of $\$44.29=\$36.908+$. $\$2580-\$36.908$
$=\$2543.09+$ *Ans.*

(13) $\$3.50\times225=\$787.50$, purchase price. $\$4\times225=\$900$:
$\$900\times183\div60=15\times183=2745$. Int.$=\$27.45$
$\$900-\$27.45=\$872.55$, proceeds of note.
$\$872.55-\$787.50=\$85.05$ gain *Ans.*

(14) $\$1+\$.07=\$1.07$: $\$535\div\$1.07=\$500$: $\$535-\$500$
$=\$35$, true discount. $\$535\times.07=\$37.45$, bank discount.
$\$37.45-\$35=\$2.45$ *Ans.*

(15) $\$.06\times4=\$.24$: $\$1+\$.24=\$1.24$: $\$1209\div\$1.24$
$=\$975$: $\$1209-\$975=\$234$, true discount.
$\$1209\times.06=\$72.54$: $\$72.54\times4=\$290.16$, bank discount.
$\$290.16-\$234=\$56.16$ *Ans.*

ARTICLE 241.

(2) Int. of \$1 for 3 mon. 3 da. (Art. 223) $=\$.0155$:
$\$1-\$.0155=\$.9845$: $\$393.80\div\$.9845=\$400$ *Ans.*

(3) Int. of \$1 for 63 da. (Art. 223) $=\$.0105$:
$\$1-\$.0105=\$.9895$: $\$500\div\$.9895=\$505.305+$ *Ans.*

REMARK.—For a full discussion of the difference between *Bank* and *true* discount, see *Ray's Higher Arithmetic.*

ARTICLE 243.—CASE I. PROFIT AND LOSS.

(3) $\$150\times.10=\$15$ *Ans.* **(4)** $\$874\times.25=\$218.50$ *Ans.*

(5) $\$500\times.12=\$60$: $\$500+\$60=\$560$ *Ans.*

(6) $\$382.50\times.04=\$15.30$: $\$382.50-\$15.30=\$367.20$ *Ans.*

ARTICLE 244.—CASE II.

(2) $\$.90\times.25=22\frac{1}{2}$: 90 cts.$+22\frac{1}{2}=\$1.12\frac{1}{2}$ *Ans.*
$\$.90\times.10=.09$: 90 cts.—9 cts.=81 cts. $\$.81$ *Ans.*

(3) $\$4.37\frac{1}{2}\times.33\frac{1}{3}=\$1.458+$: $\$4.375+\$1.458=\$5.83+$ *Ans.*
$\$4.37\frac{1}{2}\times.20=\$.87\frac{1}{2}$: $\$4.37\frac{1}{2}-\$.87\frac{1}{2}=\$3.50$ *Ans.*

(4) $\$8.50\times.20=\$1.70$: $\$8.50+\$1.70=\$10.20$:
and $\$8.50-\$1.70=\$6.80$ *Ans.*

(5) $\$300\div40=\$7.50$: $\$7.50\times.20=\$1.50$: $\$7.50+1.50$
$=\$9.00$ *Ans.* : $\$7.50-\$1.50=\$6.00$ *Ans.*

ARTICLE 245.

(6) 10cts.$+\frac{1}{10}$ of 10cts.=11cts. 15cts.$+\frac{1}{10}$ of 15cts.$=16\frac{1}{2}$cts.
20cts.$+\frac{1}{10}$ of 20cts.=22cts. 30cts$+\frac{1}{10}$ of 30cts.=33 cts.
40cts.$+\frac{1}{10}$ of 40cts.=44cts. 50cts.$+\frac{1}{10}$ of 50cts.=55 cts.
60cts.$+\frac{1}{10}$ of 60cts.=66cts.

(7) 8cts.$+\frac{1}{8}$ of 8cts.= 9cts. 12cts.$+\frac{1}{8}$ of 12cts.$=13\frac{1}{2}$cts.
16cts.$+\frac{1}{8}$ of 16cts.=18cts. 20cts.$+\frac{1}{8}$ of 20cts.$=22\frac{1}{2}$cts.

(8) 10cts $+\frac{1}{5}$ of 10cts.=12cts. 15cts.$+\frac{1}{5}$ of 15cts.=18cts.
25cts.$+\frac{1}{5}$ of 25cts.=30cts.

(9) $\$1+\frac{1}{4}$ of $\$1=\$1.25$ $\$1.20+\frac{1}{4}$ of $\$1.20=\$1.50$
$\$1.50+\frac{1}{4}$ of $\$1.50=\$1.87\frac{1}{2}$ $\$2+\frac{1}{4}$ of $\$2=\$2.50$
$\$3+\frac{1}{4}$ of $\$3=\$3.75$ $\$4+\frac{1}{4}$ of $\$4=\$5.00$
$\$6+\frac{1}{4}$ of $\$6=\$7.50$

(10) 25cts.$+\frac{1}{3}$ of 25cts.$=33\frac{1}{3}$cts. 50cts.$+\frac{1}{3}$ of 50cts.$=66\frac{2}{3}$cts.

(11) $\$2+\frac{1}{2}$ of $\$2=\$3.00$. $\$3+\frac{1}{2}$ of $\$3=\$4.50$
$\$4+\frac{1}{2}$ of $\$4=\$6$. $\$5+\frac{1}{2}$ of $\$5=\$7.50$ $\$7+\frac{1}{2}$ of $\$7.$
$=\$10.50$

ARTICLE 246.—CASE III.

(2) $\$105-\$75=\$30$: $\frac{30}{75}=\frac{2}{5}=.40=40\%$ *Ans.*

(3) $\$\ 40-\$30=\$10$: $\frac{10}{30}=\frac{1}{3}=.33\frac{1}{3}=33\frac{1}{3}\%$ *Ans.*

(4) 30 cts.—25 cts.=5 cts. : $\frac{5}{25}=\frac{1}{5}=.20=20\%$ *Ans.*

(5) $21\text{ cts.}-20\text{ cts.}=1\text{ ct.} : \frac{1}{20}=.05=5\ \%$ *Ans.*

(6) $\$9-\$8=\$1 : \frac{1}{8}=.12\frac{1}{2}=12\frac{1}{2}\ \%$ *Ans.*

(7) $30\text{ cts.}-24\text{ cts.}=6\text{ cts.} : \frac{6}{30}-\frac{1}{5}=.20=20\ \%$ *Ans.*

(8) $\$40\times40=\$1600 : \frac{704}{1600}=\frac{11}{25}=.44=44\ \%$ *Ans.*

Article 247—Case IV.

(3) $1+$.20 =$1.20 : $6÷$1.20=$5 *Ans.*

(4) $1—$.25 =$.75 : $3÷$.75=$4 *Ans.*

(5) $1+$.25 =$1.25 : $75÷$1.25=$60 *Ans.*

(6) $1—$.25 =$.75 : $75÷$.75=$100 *Ans.*

(7) $1+$.10 =$1.10 : $.22÷$1.10=$.20 *Ans.*

(8) $\$1+\$.12\frac{1}{2}=\$1.125 : \$8.10\div\$1.125=\$7.20$ *Ans.*

(9) $1—$.15 =$.85 : $1.19÷$.85=$1.40 *Ans.*

Article 248.—Promiscuous Examples.

(1) 6 cts.×30=$1.80 : $1.80×.25=$.45 *Ans.*

(2) $250÷40=$6.25, purchase price per yard. $6.25×.20 =$1.25 : $6.25+$1.25=$7.50 *Ans.*

(3) $1\text{ hhd.}=252\text{ qt.} : \$.40\times252=\$100.80 : \$100.80-\$75 =\$25.80 : \frac{25.80}{75}=.34\frac{2}{5}=34\frac{2}{5}\ \%$ *Ans.*

(4) $2428921-1918604=510317 : \frac{510317}{1918604}=.265+ =26.5+\ \%$ *Ans.*

(5) $1519467-937903=581564 : \frac{581564}{937903}=.62+ =62+\ \%$ *Ans.*

(6) $\$1+\$.10=\$1.10 : \$.42\div\$1.10=\$.38\frac{2}{11}$, purchase price. $\$.51-\$.38\frac{2}{11}=\$.12\frac{9}{11} : \frac{12\frac{9}{11}}{38\frac{2}{11}}=\frac{141}{420}=\frac{47}{140}=33\frac{4}{7}\ \%$ *Ans.*

(7) $1+$.25=$1.25 : $1÷1.25=$.80, purchase price of rye. $1×.25=$.25 : $1—$.25=$.75, selling price of wheat. $.80—$.75=$.05 *Ans.*

(8) $9.60×14=$134.40, purchase price : $134.40×.20=$26.88 : $134.40+$26.88=$161.28, what the whole must be sold for.
$14.40×5=$72.00 : $12×4=$48.00 : 14—(5+4) =5 pieces left.
$72.00+$48.00=$120.00 : $161.28—$120=$41.28, what the remaining 5 pieces must sell for.
=$41.28÷5=$8.256 *Ans.*

(9) $1—$.14=$.86 : $1.29÷$.86=$1.50, purchase price.
$1.50×.14=$.21 : $1.50+$.21=$1.71 *Ans.*

(10) $1—$.15=$.85 : $1.36÷$.85=$1.60 : $1.856—$1.60 =$.256 : $\frac{.256}{1.60}$=.16=16 % *Ans.*

(11) $1+$.12=$1.12 : $1.96÷$1.12=$1.75, purchase price.
$1.75×.16=$.28 : $1.75—$.28=$1.47 *Ans.*

(12) $1+$.16=$1.16 : $1.682÷$1.16=$1.45, purchase price.
$1.45—$1.247=$.203 : $\frac{.203}{1.45}$=.14=14 % *Ans.*

(13) $5×.25=$1.25 : $100÷$1.25=80 yd. *Ans.*

(14) $2.50×.20=$.50 : $2.50+$.50=$3.00, selling price of each cask. $3.00÷$.05=60 lb. *Ans.* The *number* of casks has nothing to do with the solution.

(15) $1+$.25=$1.25 : $1÷$1.25=$.80=purchase price per bu. $.80×.35=$.28 : $.80+$.28=$1.08=selling price per bu. $59.40÷$1.08=55 bu. *Ans.*

ARTICLE 250.—ASSESSMENT OF TAXES.

(1) $500÷$125000=.004, or 4 mills on the dollar.
$1650×.004=$6.60 *Ans.*

(2) $1.25×1258=$1572.50, amount raised on polls.
$9057.60—$1572.50=$7485.10, amount to be raised on property. $7485.10÷$534650=.014
=1 ct. 4 m. on $1 *Ans.*

(4)

$1000	is taxed	$14.00
200	“ “	2.80
40	“ “	.56
3	“ “	.042
3 polls	. .	3.75
	Ans.	$21.152

(5)

$3000	is taxed	$42.00
500	“ “	7.00
80	“ “	1.12
9	“ “	.126
4 polls		5.00
	Ans.	$55.246

ARTICLE 251.—AMERICAN DUTIES.

(1) 100+182+264=546 : Draft=1+2+3=6 lb.
546—6=540 lb. : 540×.15=81 lb. : 540—81=459 lb.
2 cts.×459=$9.18 *Ans.*

(2) 400 lb.—4 lb. draft=396 lb. : 396×.05=19.8 lb. tare :
396—19.8=376.2, net weight of each bale.
376.2×40=15048 lb. net. $.25×15048=$3762.
$3762×.20=$752.40 *Ans.*

(3) 196 lb.—2 lb. draft=194 lb. 194×.15=29.1 lb. tare :
194—29.1=164.9 net : 164.9×75=12367.5 lb. $.05×
12367.5=$618.375 *Ans.*

ARTICLE 252.—PARTNERSHIP.

(2) $\frac{300}{800}=\frac{3}{8}$: $\frac{3}{8}$ of \$232=\$87, A's share.
$\frac{500}{800}=\frac{5}{8}$: $\frac{5}{8}$ of \$232=\$145, B's share.

(3) $70+$150+$80=$300, whole stock.
$\frac{70}{300}=\frac{7}{30}$: $\frac{7}{30}$ of \$120=\$28, A's share.
$\frac{150}{300}=\frac{1}{2}$: $\frac{1}{2}$ of \$120=\$60, B's share.
$\frac{80}{300}=\frac{4}{15}$: $\frac{4}{15}$ of \$120=\$32, C's share.

(4) $200+$400+$600=$1200, whole stock.
$\frac{200}{1200}=\frac{1}{6}$, $\frac{400}{1200}=\frac{1}{3}$, $\frac{600}{1200}=\frac{1}{2}$.
$\frac{1}{6}$ of \$427.26=\$71.21, A's share : $\frac{1}{3}$ of \$427.26
=\$142.42, B's share : and $\frac{1}{2}$ of \$427.26
=\$213.63, C's share.
$427.26÷$1200=.35605=35.605 % gain. *Ans.*

(5) 1+3+5=9. $\frac{1}{9}$ of \$90=\$10 : $\frac{3}{9}=\frac{1}{3}$ of \$90=\$30 :
$\frac{5}{9}$ of \$90=\$50 *Ans.*

(6) $2+3+5+7=17$. $\frac{2}{17}$ of $\$735.93=\$86.58$:
$\frac{3}{17}$ of $\$735.93=\$129.87$: $\frac{5}{17}$ of $\$735.93=\$216.45$:
$\frac{7}{17}$ of $\$735.93=\$303.03$ *Ans.*

(7) $3+6+9+11+13+17=59$.
$\frac{3}{59}$ of $\$22361=\$1137$: $\frac{6}{59}$ of $\$22361=\$2274$:
$\frac{9}{59}$ of $\$22361=\$3411$: $\frac{11}{59}$ of $\$22361=\$4169$:
$\frac{13}{59}$ of $\$22361=\$4927$: $\frac{17}{59}$ of $\$22361=\$6443$ *Ans.*

(8) $\frac{1}{3}, \frac{3}{5}, \frac{7}{8}=\frac{40}{120}, \frac{72}{120}, \frac{105}{120}$. Since the denominators are the same, the fractions are to each other as their numerators. $40+72+105=217$. $\frac{40}{217}$ of $\$692.23=\$127.60$: $\frac{72}{217}$ of $\$692.23=\$229.68$: $\frac{105}{217}$ of $\$692.23$
$=\$334.95$ *Ans.*

ARTICLE 253.—BANKRUPTCY.

(9) $175+$500+$600+$210+$42.50+$20+$10=$1557.50.
As $1557.50 : $175 : : $934.50 : $105.00, A's share.
As $1557.50 : $500 : : $934.50 : $300.00, B's share.
As $1557.50 : $600 : : $934.50 : $360.00, C's share.
As $1557.50 : $210 : : $934.50 : $126.00, D's share.
As $1557.50 : $42.50 : : $934.50 : $25.50, E's share.
As $1557.50 : $20 : : $934.50 : $12.00, F's share.
As $1557.50 : $10 : : $934.50 : $6.00, G's share.

(10) $234+$175+$326=$735 : $492.45÷$735=$.67
= sum paid on each dollar of indebtedness.
$234×.67=$156.78 : $175×.67=$117.25 :
$326×.67=$218.42 *Ans.*

ARTICLE 254.—GENERAL AVERAGE.

(11) $\frac{48}{108}=\frac{4}{9}$; $\frac{36}{108}=\frac{1}{3}$; $\frac{24}{108}=\frac{2}{9}$. $\frac{4}{9}$ of $45=20$, A's loss.
$\frac{1}{3}$ of $45=15$, B's loss : $\frac{2}{9}$ of $45=10$, C's loss.

(12) $\$10000+\$15000=\$25000$. $\$1125\div\$25000=.04\frac{1}{2}=$
$4\frac{1}{2}$ %, general average. $\$2150\times.04\frac{1}{2}=\$96.75$, A's loss.
$\frac{3}{8}$ of $\$10000=\$3750$: $\$3750\times.04\frac{1}{2}=\$168.75$ *Ans.*

ARTICLE 255.—PARTNERSHIP WITH TIME.

(3) 23×27=621 : 21×39=819 : 621+819=1440.
$\frac{621}{1440}=\frac{69}{160}$: $\frac{819}{1440}=\frac{91}{160}$: $\frac{69}{160}$ of $\$54=\$23.28\frac{3}{4}$, A pays.
$\frac{91}{160}$ of $\$54=\$30.71\frac{1}{4}$, B pays.

(4) $300×5=$1500 : $400×8=$3200 : $500×3=$1500.
$1500+$3200+$1500=$6200.
$\frac{1500}{6200}=\frac{15}{62}$; $\frac{3200}{6200}=\frac{16}{31}$. $\frac{15}{62}$ of $\$100=\$24.19\frac{11}{31}$, A's and C's loss : $\frac{16}{31}$ of $\$100=\$51.61\frac{9}{31}$, B's loss.

(5) 6×30=180 : 5×40=200 : 8×28=224.
180+200+224=604: $\frac{180}{604}=\frac{45}{151}$; $\frac{200}{604}=\frac{50}{151}$; $\frac{224}{604}=\frac{56}{151}$.
$\frac{45}{151}$ of $\$18.12=\$5.40$, A : $\frac{50}{151}$ of $\$18.12=\$6$, B :
$\frac{56}{151}$ of $\$18.12=\$6.72$, C.

(6) A $300× 8=$2400, $300+100=$400,
$400×8=$3200, $2400+3200=$5600.
B $600×10=$6000, $600—300=$300,
$300×6=$1800, $6000+1800=$7800.
C $500×12=$6000, $500+400=$900,
$900× 4=$3600, $6000+3600=$9600.
$5600+7800+9600=$23000.
As 23000 : 5600 : : $759 : $184.80, A's.
23000 : 7800 : : $759 : $257.40, B's.
23000 : 9600 : : $759 : $316.80, C's.

(7) $800×12=$9600 : $500×12=$6000 : 12 mon.—7 mon. =5 mon. : $9600—$6000=3600 : $3600÷5=$720 *Ans.*

EQUATION OF PAYMENTS.

ARTICLE 257.

(2) $2×4=$8
6×8=48
$8) 56(7 mon. *Ans.*

(3) $8×5=$40
4×8= 32
$12) 72(6 mon. *Ans.*

(4) \$250×2=\$500
500×5=2500
750×8=6000
\$1500) 9000(6 mon. *Ans.*

(5) \$100× 6=\$600
75× 8= 600
125×12=1500
\$300) 2700(9 mon. *Ans.*

(6) $\frac{1}{5}$ of \$200=\$40.00
$\frac{2}{5}$ of 200= 80.00
\$40× 0= 0
80× 5=400
80× 10=800
\$200) 1200(6 mon. *Ans.*

ARTICLE 258.

(2) Counting from April 2d, it is 90 days to the first payment, and 150 days to the second.

\$200× 90=\$18000
\$300×150= 45000
\$500) 63000(126 da. from April 2d, which will give Aug. 6th *Ans.*

ARTICLE 259.

(8) \$103÷\$1.03=\$100
\$106÷\$1.06=\$100
\$103+ 106=\$209
\$209— \$200=\$ 9
\$200× .06=\$ 12
\$9÷\$12=.75 yr.=9 mon. *Ans.*

ALLIGATION MEDIAL.

ARTICLE 260.

(2) 6 lb. at 3 cts.=18 cts.
4 lb. at 8 cts.=32 cts.
10 lb. cost 50 cts.
50 cts.÷10=5 cts. *Ans.*

(3) 25 lb. at 12 cts.=\$3.00
25 lb. at 18 cts.= 4.50
40 lb. at 25 cts.=10.00
90 lb. cost \$17.50
\$17.50÷90=\$.19$\frac{4}{9}$ *Ans.*

(4) 3 gal. cost $0.00
12 gal. at 50= 6.00
15 gal. cost $6.00
$6.00÷15=$.40 *Ans.*

(5) 10 at $3=$30.00
12 at 4= 48.00
8 at 9= 72.00
30 worth $150.00
$150÷30=$5 *Ans.*

(6) 6 to 10= 4 hr. : 63°× 4=252°
10 to 1= 3 : 70°× 3=210°
1 to 3= 2 : 75°× 2=150°
3 to 7= 4 : 73°× 4=292°
7 to 6=11 hr. : 55°×11=605°
24)1509°($62\frac{7}{8}$° *Ans.*

ANALYSIS.

Remarks.—The subject of analysis is one of the most important in Arithmetic. With a *thorough* knowledge of the elementary rules, fractions, and analysis, nearly all the operations that are purely arithmetical may be solved without the aid of special rules; because these rules are mostly derived from *analytical* solutions of particular questions. The pupil, however, who desires to understand analysis fully, and apply it readily, must first make himself well acquainted with Mental Arithmetic. See Ray's Arithmetic, 2d Book.

Article 261.

(2) $36 **(3)** $24 **(5)** 7 bl. **(6)** 10 lb.
(7) 10 yd. **(9)** $7\frac{1}{2}$ yd. **(10)** 4. **(11)** $4\frac{3}{8}$.
(13) 12 : 20. **(14)** 28 : 27. **(16)** 10. **(17)** $8\frac{1}{4}$.
(19) 18. **(20)** 28. **(23)** 16 cts.

(24) If 1-fifth of a bushel cost 14 cents, 1 bushel will cost 5 times 14 cents, which are 70 cents; and $\frac{9}{10}$ of a bushel will cost $\frac{9}{10}$ of 70 cents, which are 63 cents. *Ans.*

(25) 1-eighth will cost 1-third of 3-eighths; if $\frac{3}{8}$ cost 21 cents, $\frac{1}{8}$ will cost $\frac{1}{3}$ of 21 cents, which is 7 cents, and if $\frac{1}{8}$ cost 7 cents, 1 yard, or 8-eighths, will cost 8 times 7 cents, which are 56 cents; then $\frac{6}{7}$ of a yard will cost $\frac{6}{7}$ of 56 cents, which are 48 cents. *Ans.*

ARTICLE 263.

(2) \$2.16÷24=9 cts. : 9 cts.×23=\$2.07 *Ans.*

(3) \$32.50÷13=\$2.50 : \$2.50×14=\$35 *Ans.*

(4) \$8÷4=\$2 : \$24÷\$2=12 yd. *Ans.*

(5) \$20.67÷159=\$.13 : \$34.71÷\$.13=267 yd. *Ans.*

(6) \$147÷34=\$4$\frac{11}{34}$: \$4$\frac{11}{34}$×9=\$38$\frac{31}{34}$ *Ans.*

(8) 25 da.×17=425 da. : 425 da.÷10=42$\frac{1}{2}$ da. *Ans.*

(9) If 6 men can build a wall in 10 days, it will require 10 times 6=60 men to build it in 1 day, and 1-fifteenth of 60 men=4 men, to build it in 15 days. *Ans.*

(10) If 5 men eat a barrel of flour in 12 days, 1 man will eat a barrel in 5 times 12 days, which are 60 days, and 4 men will eat a barrel in 1-fourth of the time of 1 man, that is, in $\frac{1}{4}$ of 60 days=15 da. *Ans.*

(11) 10 hr.×7=70 hr. to do the work: 70 hr.÷12 hr.=5$\frac{5}{6}$ da. of 12 hr. each. *Ans.*

(12) If 18 men reap 72 acres in 7 days, 1 man would reap it in 7×18=126 days, and 8 men in $\frac{1}{8}$ of 126=15$\frac{3}{4}$ da. *Ans.*

(13) Each 9 sheep are worth 5 hogs: in 54 sheep there are 6 times 9 sheep; then 54 sheep are worth 6 times 5 hogs, which are 30 hogs. *Ans.*

(14) In 42 gal. of wine there are 14 times 3 gal. of wine; hence 42 gal. of wine are worth 14 times 7 gal. of cider =98 gal. *Ans.*

(15) If a 3 cent loaf weigh 8 oz., when flour is \$4 a barrel, if the flour were \$1 a barrel it should weigh 8 oz.×4 =32 oz.; and when flour is \$5 a barrel, it should weigh 1-fifth of 32 oz.=6$\frac{2}{5}$ oz. *Ans.*

(16) If 3 stacks of hay keep 12 horses 5 mon., they would keep 1 horse 5 mon.×12=60 mon., and 20 horses 1-twentieth of 60 mon.=3 mon. *Ans.*

(17) If $\frac{2}{5}$ of a yard cost 40 cts., $\frac{1}{5}$ of a yard will cost $\frac{1}{2}$ of 40 cts.=20 cts., and $\frac{3}{5}$ will cost 3 times 20 cts.=60 cts. *Ans.*

(18) If $\frac{1}{3}$ yd. cost \$2, one yard will cost 3 times \$2=\$6, and $\frac{3}{4}$ yd. will cost $\frac{3}{4}$ of \$6=\$4.50. *Ans.*

(19) If $\frac{5}{7}$ of a tun cost \$4.25; one-seventh will cost $\frac{1}{5}$ of \$4.25=\$.85, and a tun will cost 7 times \$.85=\$5.95 : then $\frac{11}{17}$ of a tun will cost $\frac{11}{17}$ of \$5.95=\$3.85 *Ans.*

(20) If $\frac{9}{11}$ of a cord cost \$2.52, one-eleventh will cost $\frac{1}{9}$ of \$2.52=\$.28, and a cord will cost 11 times \$.28=\$3.08 : then $\frac{13}{14}$ of a cord will cost $\frac{13}{14}$ of \$3.08=\$2.86 *Ans.*

(21) If $\frac{3}{7}$ lb. cost \$$\frac{1}{4}$, one-seventh will cost $\frac{1}{3}$ of \$$\frac{1}{4}$=\$$\frac{1}{12}$, and a lb. will cost 7 times \$$\frac{1}{12}$=$\frac{7}{12}$; then $\frac{8}{9}$ lb. will cost $\frac{8}{9}$ of \$$\frac{7}{12}$=\$$\frac{14}{27}$ *Ans.*

Article 264.

(23) If 2 men can earn \$16 in 4 days, 1 man will earn $\frac{1}{2}$ of \$16=\$8 in 4 days, and in 1 day he will earn $\frac{1}{4}$ of \$8 =\$2: then if 1 man earn \$2 in 1 day, 7 men will earn 7 times \$2=\$14, in 1 day; and in 3 days they will earn 3 times \$14=\$42 *Ans.*

(24) If 2 men can build 12 rd. in 9 days, 1 man can build $\frac{1}{2}$ of 12 rd.=6 rd. in 9 days, and in 1 day he can build $\frac{1}{9}$ of 6 rd.=$\frac{2}{3}$ rd. : then if 1 man can build $\frac{2}{3}$ rd. in 1 day, 7 men can build 7 times $\frac{2}{3}$ rd.=$4\frac{2}{3}$ rd. in 1 day, and in 6 days, they can build 6 times $4\frac{2}{3}$ rd.=28 rd. *Ans.*

(25) If 11 A. can be plowed in 5 da. with 15 oxen, it would require 15×5=75 oxen to plow 11 A. in one day; and, since 33 A.=3 times 11 A., it would require 75 oxen×3 =225 oxen to plow 33 A. in 1 day; and to plow 33 A. in 9 days, it would take $\frac{1}{9}$ of 225 oxen=25 oxen *Ans.*

Article 265.

(26) Since the first pipe *fills* 41 gal. in 1 hr., and the second *empties* 6 gal. in the same time, therefore there are left in, 41—6=35 gal. per hour; it will therefore require $\frac{250}{35}=\frac{50}{7}$ hr.=7 hr. 8 min. $34\frac{2}{7}$ sec. to fill the cistern.

(27) 600÷8=75 gal. filled by A in 1 hr. : 600÷12=50 gal. filled by B in 1 hr. : then 75+50=125 gal. filled by both in 1 hr.; and $\frac{600}{125}=\frac{24}{5}=4\frac{4}{5}$ hr. *Ans.*

(28) $900\div10=90$ gal. *filled* by A in 1 hr. $900\div12=75$ gal. *emptied* by B in 1 hr. : then $90-75=15$ gal. left in per hr., and $900\div15=60$ hr. to fill the cistern.

(30) A does $\frac{1}{20}$, B $\frac{1}{15}$, and C $\frac{1}{12}$ in 1 day, hence in 1 day they all do $\frac{1}{20}+\frac{1}{15}+\frac{1}{12}=\frac{3}{60}+\frac{4}{60}+\frac{5}{60}=\frac{12}{60}=\frac{1}{5}$ in one day, and therefore all do it in $1\div\frac{1}{5}=5$ days. *Ans.*

(31) Reasoning as in the preceding example, A does $\frac{1}{12}$, B $\frac{1}{15}$, C $\frac{1}{18}$, and D $\frac{1}{24}$ in 1 day, and in 1 day they all do $\frac{1}{12}+\frac{1}{15}+\frac{1}{18}+\frac{1}{24}=\frac{30}{360}+\frac{24}{360}+\frac{20}{360}+\frac{15}{360}=\frac{89}{360}$; hence it will require $1\div\frac{89}{360}=4\frac{4}{89}$ days to do the whole.

(32) $\frac{1}{8}+\frac{1}{9}=\frac{9}{72}+\frac{8}{72}=\frac{17}{72}$, and $1\div\frac{17}{72}=4\frac{4}{17}$ da. *Ans.*
Or, suppose the meadow to consist of 72 parts, then A mows $\frac{1}{8}$ of $72=9$ parts in one day, and B $\frac{1}{9}$ of $72=8$ parts in one day : $9+8=17$ parts both do in one day;
$72\div17=4\frac{4}{17}$ da. *Ans.*

(33) Since A and B together, mow it in 12 days, they mow $\frac{1}{12}$ in one day, and since A can mow it in 20 days, he can mow $\frac{1}{20}$ in one day, hence B can mow $\frac{1}{12}-\frac{1}{20}=\frac{5}{60}-\frac{3}{60}=\frac{2}{60}=\frac{1}{30}$ in one day; hence he can mow it in $1\div\frac{1}{30}=30$ da. *Ans.*

(34) Since both eat it in 6 days, they eat $\frac{1}{6}$ in 1 day, and since the woman eats it in 15 days, she eats $\frac{1}{15}$ in 1 day; hence the man eats $\frac{1}{6}-\frac{1}{15}=\frac{5}{30}-\frac{2}{30}=\frac{3}{30}=\frac{1}{10}$ in 1 day, therefore he will eat in $1\div\frac{1}{10}=10$ da. *Ans.*

(35) The first will empty $\frac{1}{4}$, the second $\frac{1}{8}$, and the third $\frac{1}{15}$ in 1 minute, hence they will all empty $\frac{1}{4}+\frac{1}{8}+\frac{1}{15}=\frac{30}{120}+\frac{15}{120}+\frac{8}{120}=\frac{53}{120}$ in 1 minute; and $1\div\frac{53}{120}=\frac{120}{53}=2$ min. $15\frac{45}{53}$ sec. *Ans.*

(36) Since A can mow $\frac{3}{5}$ in 6 days, he can mow $\frac{1}{5}$ in $\frac{1}{3}$ of 6 days, which is 2 days, and since he can mow $\frac{1}{5}$ in 2 days, he can mow the whole in 5 times 2 days=10 days; therefore he can mow $\frac{1}{10}$ in 1 day. Again, since B can mow $\frac{5}{7}$ in 10 days, he can mow $\frac{1}{7}$ in $\frac{1}{5}$ of 10 days, which is 2 days, and since he can mow $\frac{1}{7}$ in 2 days, he can

mow the whole in 7 times 2 days=14 days; therefore he can mow $\frac{1}{14}$ in 1 day.
$\frac{1}{10}+\frac{1}{14}=\frac{7}{70}+\frac{5}{70}=\frac{12}{70}=\frac{6}{35}$, the part both mow in one day, hence $1\div\frac{6}{35}=5\frac{5}{6}$ days, the time in which both can do it.

(38) \$3000—\$500=\$2500 : \$2500÷2=\$1250, less share, and \$1250+\$500=\$1750, greater share.

(39) The wife received \$800 more than the son, and the son \$300 more than the daughter; hence, the wife received \$800+\$300=\$1100 more than the daughter.
\$300+\$1100=\$1400, what the wife and son received more than twice the daughter's.
\$3500—\$1400=\$2100 : \$2100÷3=\$700= daughter's share : \$700+\$300=\$1000, son's share : \$1000+\$800 =\$1800, wife's share.

(41) In this case the minute hand must *gain* 25 min. on the hour hand. Now, according to the solution of Example 40, in the Arithmetic, $\frac{12}{11}$ of this 25 min. is the distance over which the minute hand must move to overtake the hour hand. $\frac{12}{11}$ of $25=\frac{300}{11}=27\frac{3}{11}$ min. *Ans.*

(42) When the hour hand is at VIII, the minute hand is at XII. If the hour hand did not move, the minute hand would have to pass over 10 minutes to be opposite to the hour hand, but since the hour hand does move, the minute hand must *gain* 10 minutes before it is opposite to the hour hand. It is then required to find what distance the minute hand must move to *gain* 10 minutes on the hour hand. Reasoning as in the Solution to Example 40, we find $\frac{12}{11}$ of $10=10\frac{10}{11}$ min. *Ans.*

(43) $\frac{5}{12}$ of \$3276=\$1365 : Since $\frac{5}{12}$ of what A put in, was equal to $\frac{5}{9}$ of what B put in, $\frac{1}{5}$ of \$1365=\$273, is $\frac{1}{9}$ of what B put in; hence \$273×9=\$2457, what B put in. $\frac{11}{27}$ of \$2457=\$1001; \$3276—\$1001=\$2275, which is $\frac{5}{4}$ of what C put in; hence $\frac{1}{5}$ of \$2275=\$455=$\frac{1}{4}$ of what C put in; hence C put in \$455×4=\$1820. Since A received a sum the $\frac{6}{5}$ of which was equal to the amount he put in, $\frac{1}{6}$ of the amount he put in, multiplied by 5, must equal his share; $\frac{1}{6}$ of $3276=$546, and $546×5 =$2730, A's share. $\frac{6}{5}$ of 1820=$2184, C's gain. $7000— ($2730+$2184) =$2086, B's gain.

ARTICLE 266.

(45) By taking 1 lb. at 4 cts. I gain 1 ct.; and by taking 1 lb. at 8 cts., I lose 3 cts.; hence, in order to equalize the gains and losses, as often as 1 lb. at 8 cts. is taken, there must be taken 3 lb. at 4 cts.

(46) By taking 1 lb. at 3 cts. there is a gain of 3 cts., and by taking 1 lb. at 8 cts. there is a loss of 2 cts.; hence, to equalize the gains and losses, there must be taken 2 lb. at 3 cts., which will give a gain of 6 cts., and 3 lb. at 8 cts., which will give a loss of 6 cts.

(48) On each lb. at 11 cts. there is a loss of 1 ct., hence on 10 lb. there will be a loss of 1 ct.×10=10 cts. On each lb. at 8 cts. there is a gain of 2 cts.; hence, if 1 lb. gains 2 cts., it is merely required to find how many lb. must be taken to gain 10 cts., which is evidently as many as 2 cts. is contained times in 10 cts., that is, 5 lb. *Ans.*

ARTICLE 267.

(51) $\frac{1}{3}-\frac{1}{4}=\frac{1}{12}$; hence $\frac{1}{12}$ of the number is 8, and the number is 12 times 8=96 *Ans.*

(52) $\frac{7}{10}-\frac{3}{5}=\frac{1}{10}$; hence $\frac{1}{10}$ of the number is 7, and the number is 10 times 7=70 *Ans.*

(53) $\frac{1}{3}+\frac{1}{5}=\frac{8}{15}$, and $1-\frac{8}{15}=\frac{7}{15}$, the part of his money left. Then $\frac{7}{15}$ of his money is $35, and $\frac{1}{15}$ is $\frac{1}{7}$ of 35, which is $5, and 15-fifteenths, or the whole, is 15 times $5, which is $75 *Ans.*

(54) $1+\frac{1}{2}+\frac{1}{3}+\frac{3}{5}=\frac{30}{30}+\frac{15}{30}+\frac{10}{30}+\frac{18}{30}=\frac{73}{30}$; hence $\frac{73}{30}$=73, and $\frac{1}{30}$ is $\frac{1}{73}$ of 73, which is 1, and $\frac{30}{30}$, or the whole, is 30 times 1=30 *Ans.*

(55) $1-\frac{4}{9}=\frac{5}{9}$, part remaining; he then had left 20+15=35 cts.; if $\frac{5}{9}$ is 35 cts., $\frac{1}{5}$ of 35 cts., which is 7 cts., is $\frac{1}{9}$, and the whole is 9 times 7 cts.=63 cts. *Ans.*

(56) $1-\frac{2}{3}=\frac{1}{3}$; $\frac{3}{5}$ of $\frac{1}{3}=\frac{1}{5}$: $\frac{1}{3}-\frac{1}{5}=\frac{2}{15}$, part left; then $\frac{2}{15}=10$ cts.; and $\frac{1}{2}$ of 10 cts., which is 5 cts, is $\frac{1}{15}$; hence $\frac{15}{15}$, or the whole, is 15 times 5 cts.=75 cts. *Ans.*

(57) Of each 4 days he worked 3, earning \$6, and idled 1 day, forfeiting \$1; hence, for each 3 days he worked he received \$5, and since he received \$25, which are 5 times \$5, he worked 5 times 3 da.=15 da. *Ans.*

(58) $\frac{1}{2}+\frac{1}{4}+\frac{1}{5}=\frac{10}{20}+\frac{5}{20}+\frac{4}{20}=\frac{19}{20}$; $1-\frac{19}{20}=\frac{1}{20}=$ part bearing pears; hence $\frac{1}{20}$ is 4, and $\frac{20}{20}$, or the whole, are 20 times 4=80 *Ans.*

(59) As many, (1,) as many more, (1,) half as many more, ($\frac{1}{2}$,) and one-fourth as many more, ($\frac{1}{4}$,) are equal to $1+1+\frac{1}{2}+\frac{1}{4}=2\frac{3}{4}=\frac{11}{4}$; then since $\frac{11}{4}$ of the number are 110, one-fourth will be $\frac{1}{11}$ of 110, which is 10, and the whole, or 4-fourths, will be 4 times 10=40 *Ans.*

(60) $1-\frac{1}{4}=\frac{3}{4}$,=part left at close of 1st day.
$\frac{1}{4}$ of $\frac{3}{4}=\frac{3}{16}$; $\frac{3}{4}-\frac{3}{16}=\frac{9}{16}$=part left at close of 2d day.
$\frac{1}{4}$ of $\frac{9}{16}=\frac{9}{64}$; $\frac{9}{16}-\frac{9}{64}=\frac{27}{64}$=part left at close of 3d day.
$\frac{1}{4}$ of $\frac{27}{64}=\frac{27}{256}$; $\frac{27}{64}-\frac{27}{256}=\frac{81}{256}$=part left at close of 4th day.
Then since $\frac{81}{256}$=\$1.62, 1-eighty-first part of \$1.62, which is 2 cts., is $\frac{1}{256}$ part of the whole; hence $\frac{256}{256}$, or the whole, are 256 times 2 cts.=\$5.12 *Ans.*

(61) $1+\frac{1}{2}=\frac{3}{2}$=part of capital at close of 1st yr.
$\frac{3}{5}$ of $\frac{3}{2}=\frac{9}{10}$; $\frac{3}{2}+\frac{9}{10}=\frac{12}{5}$=part of capital at close of 2d yr.
$\frac{1}{4}$ of $\frac{12}{5}=\frac{3}{5}$; $\frac{12}{5}-\frac{3}{5}=\frac{9}{5}$=part of capital at close of 3d yr.
Then since $\frac{9}{5}$=\$3375, one-ninth of \$3375, which is \$375, will be one-fifth of the capital, and $\frac{5}{5}$, or the whole, will be 5 times \$375=\$1875 *Ans.*

EXCHANGE OF CURRENCIES.

ARTICLE 269.

(1) £5×20+3 s.=103 s. : 103 s.×12+2 d.=1238 d. :
1238 d.×4+1 far.=4953 far. *Ans.*

(2) 8675 far.÷4=2168$\frac{3}{4}$ d. : 2168$\frac{3}{4}$ d.÷12=180 s. 8$\frac{3}{4}$ d. :
180 s.÷20=£9. *Ans.* £9 8$\frac{3}{4}$ d.

(3)

	£	s.	d.
	3	0	$6\frac{1}{4}$
	5	10	$4\frac{1}{2}$
	2	15	$1\frac{1}{4}$
Ans.	11	6	0

(4)

	£	s.	d.
	17	6	5
	8	5	$11\frac{1}{2}$
Ans.	9	0	$5\frac{1}{2}$

(5)

	£	s.	d.
	3	12	$2\frac{1}{2}$
			8
Ans.	28	17	8

(6)

	£	s.	d.
6)	25	0	$10\frac{1}{2}$
Ans.	4	3	$5\frac{3}{4}$

(7) £.625×20=12.5 s. : .5 s.×12=6 d. *Ans.* 12 s. 6 d.

(8) £.796875×20=15.9375 s. : .9375 s.×12=11.25 d. : .25 d.×4=1 far.=$\frac{1}{4}$ d. *Ans.* 15 s. $11\frac{1}{4}$ d.

(9) 7 s. 6 d.=90 d. : £1=240 d. : $\frac{90}{240}=\frac{3}{8}$=£.375 *Ans.*

(10) 8 s. 9 d.=105 d. : £1=240 d. : $\frac{105}{240}=\frac{7}{16}$=£.4375 *Ans.*

ARTICLE 270.

(11) £75 10 s.=£75.5 : £75.5×.04=£3.02 : £3.02×$2\frac{1}{2}$ =£7.55 : £.55×20=11 s. *Ans.* £7 11 s.

(12) 12 s. 6d.=£.625 : £85.625×.06=£5.1375 : £5.1375× $1\frac{3}{4}$ (1 yr. 9 mon.)=£8.990625 : £.990625×20 =19.8125 s. : .8125 s.×12=9.75 d. : .75 d.×4=3 far., or $\frac{3}{4}$ d. *Ans.* £8 19 s. $9\frac{3}{4}$ d.

ARTICLE 271.

(4) £25×4.84=$121 *Ans.*

(5) 8 s.=£.4. £15.4×4.84=$74.536 *Ans.*

(6) 15 s. 9 d.=£.7875. £36.7875×4.84=$178.05+ *Ans.*

(7) $179.08÷$4.84=£37 *Ans.*

(8) $124.388÷$4.84=£25.7=£25 14 s. *Ans.*

(9) $1000÷$4.84=£206.61157+=£206 12 s. $2\frac{3}{4}$ d.+ *Ans.*

ARTICLE 272.

(2) £150×$4\frac{4}{9}$=\$$666\frac{2}{3}$: \$$666\frac{2}{3}$×.08=\$$53.33\frac{1}{3}$=\$$53\frac{1}{3}$ prem. \$$666\frac{2}{3}$+\$$53\frac{1}{3}$=\$720 *Ans.*

(3) 10 s.=£.5 £80.5×$4\frac{4}{9}$=\$357.777+ : \$357.777×$.09\frac{1}{2}$= \$33.988+ prem. : \$357.777+\$33.988=\$391.76+ *Ans.*

ARTICLE 273.

(1) \$120.50×$\frac{3}{10}$=£36.15=£36 3 s. *Ans.*

(2) \$ 75.25×$\frac{2}{5}$=£30.1 =£30 2 s. *Ans.*

(3) \$98×$\frac{3}{8}$=£36.75=£36 15 s. *Ans.*

(4) £30 15 s.=£30.75 : £30.75÷$\frac{3}{10}$=\$102.50 *Ans.*

(5) £25 17 s.=£25.85 : £25.85÷$\frac{2}{5}$=\$64.625 *Ans.*

(6) £29 8 s.=£20.4 : £29.4 ÷$\frac{7}{30}$=\$126.00 *Ans.*

ARTICLE 274.

Ex. 1166 fr.÷5.30 fr.=\$220 *Ans.*

ARTICLE 276.—DUODECIMALS.

(2)

	ft.	in.	
	5	3	
	1	5	
	5	3	″
	2	2	3
Ans.	7	5	3

(3)

	ft.	in.	
	5	7	
	1	10	
	5	7	″
	4	7	10
Ans.	10	2	10

(4)

	ft.	in.	″	
	8	6	9	
	7	3		
	59	11	3	‴
	2	1	8	3
Ans.	62	0	11	3

(5)

	ft.	in.	″	
	8	4	6	
	2	7	4	
	16	9	0	‴
	4	10	7	6
		2	9	6
Ans.	21	10	5	0

(6)

	ft.	in.	″		
	4	5	6		
	2	3	5		
	8	11	0	‴	
	1	1	4	6	⁗
		1	10	3	6
Ans.	10	2	2	9	6

ARTICLE 279.—INVOLUTION.

(1) 1—4—9—16—25—36—49—64—81—100—121.

(2) 1—8—27—64.

(3) $\frac{1}{4}$—$\frac{1}{9}$—$\frac{4}{9}$—$\frac{1}{16}$.

(4) $\frac{1}{8}$—$\frac{1}{27}$—$\frac{8}{27}$—$\frac{27}{64}$.

(5) 16—32—81.

(6) $65\times65=4225$ *Ans.*

(7) $25\times25\times25=15625$ *Ans.*

(8) $16\frac{1}{2}=\frac{33}{2}$. $\frac{33}{2}\times\frac{33}{2}=\frac{1089}{4}=272\frac{1}{4}$ *Ans.*

(9) $12\frac{1}{2}=\frac{25}{2}$. $\frac{25}{2}\times\frac{25}{2}\times\frac{25}{2}=\frac{15625}{8}=1953\frac{1}{8}$ *Ans.*

(10) $13\times13\times13\times13=28561$ *Ans.*

(11) $\frac{2}{3}\times\frac{2}{3}\times\frac{2}{3}\times\frac{2}{3}\times\frac{2}{3}=\frac{32}{243}$ *Ans.*

(12) $9\times9\times9\times9\times9\times9=531441$ *Ans.*

(13) $.025\times.025\times.025\times.025=.000000390625$ *Ans.*

(14) $14^3=14\times14\times14=2744$ *Ans.*

(15) $19^4=19\times19\times19\times19=130321$ *Ans.*

(16) $2\frac{1}{3}^5=\frac{7}{3}\times\frac{7}{3}\times\frac{7}{3}\times\frac{7}{3}\times\frac{7}{3}=\frac{16807}{243}=69\frac{40}{243}$ *Ans.*

(17) $.09^6=.09\times.09\times.09\times.09\times.09\times.09=.000000531441$ *Ans.*

ARTICLE 286.

(2)
```
 529(20+3=23 Ans.
 400
 ----
20|129
 2|129
--|---
40|
 3|
--|
43|
```

ARTICLE 287.

```
     . .                 . .                . . .
(6) 625(25 Ans.   (7) 6561(81 Ans.   (8) 390625(625 Ans.
     4                   64                 36
    ---                 ----               ---
 45)225             161)161            122)306
    225                 161                244
    ---                 ---                ----
                                      1245)6225
                                           6225
                                           ----
     . . . .
(9)  1679616(1296 Ans.
     1
     ----
  22)67
     44
     ----
 249)2396
     2241
     -----
2586)15516
     15516
     -----
```

```
      . . . .
(10)  5764801(2401 Ans.
      4
      ---
   44)176
      176
      ----
 4801)4801
      4801
      ----
```

```
      . . . .
(11)  43046721(6561 Ans.
      36
      ---
  125)704
      625
      ----
 1306)7967
      7836
      -----
13121)13121
      13121
      -----
```

```
      . . . . .
(12)  987656329(31427 Ans.
      9
      ---
   61)87
      61
      ----
  624)2665
      2496
      -----
 6282)16963
      12564
      ------
62847)439929
      439929
      ------
```

```
      . . . . .
(13)  289442169(17013 Ans.
      1
      ---
   27)189
      189
      -----
 3401)4421
      3401
      ------
34023)102069
      102069
      ------
```

```
      .  . .
(14)  234.09(15.3 Ans.
      1
      ---
   25)134
      125
      ----
  303)909
      909
      ---
```

(15) $\dot{1}4\dot{5}.2\dot{0}2\dot{5}$(12.05 *Ans.*
1
22)45
44
2405)12025
12025

(16) $.0\dot{1}9\dot{6}$(.14 *Ans.*
1
24)96
96

(17) $\dot{1}.0\dot{0}8\dot{0}1\dot{6}$(1.004 *Ans.*
1
2004)008016
8016

(18) $.0\dot{0}8\dot{2}2\dot{6}4\dot{9}$(.0907 *Ans.*
81
1807)12649
12649

(19) $\sqrt{25}=5$, $\sqrt{729}=27$, $\sqrt{\frac{25}{729}}=\frac{5}{27}$ *Ans.*

(20) $\frac{847}{1183}=\frac{121}{169}$, $\sqrt{121}=11$, $\sqrt{169}=13$, $\sqrt{\frac{121}{169}}=\frac{11}{13}$ *Ans.*

(21) $30\frac{1}{4}=\frac{121}{4}$, $\sqrt{\frac{121}{4}}=\frac{11}{2}=5\frac{1}{2}$ *Ans.*

(22) $\dot{1}0$(3.162277+ *Ans.*
9
61)100
61
626)3900
3756
6322)14400
12644
63242)175600
126484
632447)4911600
4427129
6324547)48447100
44271829

(23) $\dot{2}$(1.41421+ *Ans.*
1
24)100
96
281)400
281
2824)11900
11296
28282)60400
56564
282841)383600
282841

(24) $\frac{2}{3}=.\dot{6}6\dot{6}6\dot{6}6+(.81649+$ *Ans.*
64
161)266
161
1626)10566
9756
16324)81066
65296
163289)1577066
1469601

(25) $6\frac{2}{5}=\dot{6}.\dot{4}(2.5298+$ *Ans.*
4
45)240
225
502)1500
1004
5049)49600
45441
50588)415900
404704

(26) $\sqrt{6241}=79$ *Ans.*

ARTICLE 288.

(1) $16=2\times2\times2\times2$: $\sqrt{16}=2\times2=4$ *Ans.*
(2) $36=2\times2\times3\times3$: $\sqrt{36}=2\times3=6$ *Ans.*
(3) $100=2\times2\times5\times5$: $\sqrt{100}=2\times5=10$ *Ans.*
(4) $225=3\times3\times5\times5$: $\sqrt{225}=3\times5=15$ *Ans.*
(5) $\sqrt{(16\times25)}=4\times5=20$ *Ans.*
(6) $\sqrt{(36\times49)}=6\times7=42$ *Ans.*
(7) $\sqrt{(64\times81)}=8\times9=72$ *Ans.*
(8) $\sqrt{(121\times25)}=11\times5=55$ *Ans.*

ARTICLE 290.

(1) $30^2=900$
$40^2=1600$
$\sqrt{2500}=50$ *Ans.*

(2) $81^2=6561$
$108^2=11664$
$\sqrt{18225}=135$ *Ans.*

(3) $100^2=10000$
$60^2=3600$
$\sqrt{6400}=80$ *Ans.*

(4) $45^2=2025$
$60^2=3600$
$\sqrt{5625}=75$ *Ans.*

(5) $36^2=1296$
$24^2=576$
$\sqrt{720}=26.83+$ *Ans.*

(6) $60^2=3600$
$37^2=\underline{1369}$
$\sqrt{2231}=47.2334+$=width of street from foot of ladder on one side.
$60^2=3600$
$23^2=\underline{\ 529}$
$\sqrt{3071}=\underline{55.4166+}$=width of street from foot of ladder on the other side.
Ans. 102.65

(7) $600^2=360000$
$140^2=\underline{\ 19600}$
$\sqrt{340400}=583.43+$
$100\div2=\underline{\ 50.}$
Ans. 533.43+

(8) $20^2=400$
$16^2=\underline{256}$
656
Square of base$=656$
$12^2=\text{per.}^2=\underline{144}$
800
$\sqrt{800}=28.28+$ *Ans.*

The square root of 656, will give the length of the diagonal line joining opposite corners of the floor of the room: this is the base of the triangle, of which the hypotenuse is required.

ARTICLE 291.

(1) $\sqrt{4096}=64$ *Ans.*

(2) $4\times4=16$: $16\times9=144$: $\sqrt{144}=12$ *Ans.*
Or, $\sqrt{(16\times9)}=4\times3=12$ *Ans.*

(3) $\sqrt{43560}=208.71+$ ft., side of acre.
$43560\div2=21780$: $\sqrt{21780}=147.58+$ ft., side of half A.
$43560\div4=10890$: $\sqrt{10890}=104.35+$ ft., side of qr. A.

(4) 10 A.=1600 P.
$12\frac{1}{2}$ A.=$\underline{2000}$
$\sqrt{3600}=60$ *Ans.*

ARTICLE 295.

$\dot{1}72\dot{8}$)10+2=12 root.
1000
10×10×3=300 | 728
10× 2×3= 60
2× 2 = 4
364 | 728

ARTICLE 296.

(5) $9\dot{1}12\dot{5}$(45 *Ans.*
64
| 27125
4×4×300=4800
4×5× 30= 600
5×5 = 25
5425 | 27125

(6) $19\dot{5}11\dot{2}$(58
125 [*Ans.*
| 70112
5×5×300=7500
5×8× 30=1200
8×8 = 64
8764 | 70112

(7) $91\dot{2}67\dot{3}$(97 *Ans.*
729
| 183673
9×9×300=24300
9×7× 30= 1890
7×7 = 49
26239 | 183673

(8) $\dot{1}22\dot{5}04\dot{3}$(107
1 [*Ans.*
1× 1×300= 300 | 225
10×10×300=30000 | 225043
10× 7× 30= 2100
7× 7 = 49
32149 | 225043

(9) $1\dot{3}31\dot{2}05\dot{3}(237$ *Ans.*

$$\begin{array}{r|l}
 & 8 \\ \cline{2-2}
2\times2\times300=1200 & 5312 \\
2\times3\times\ 30=\ \ 180 & \\
3\times3\qquad\quad=\ \ \ \ \ 9 & \\ \cline{1-1}
1389 & 4167 \\ \hline
23\times23\times300=158700 & 1145053 \\
23\times\ 7\times\ 30=\ \ \ 4830 & \\
7\times7\qquad\quad=\ \ \ \ \ \ 49 & \\ \cline{1-1}
163579 & 1145053 \\ \cline{2-2}
\end{array}$$

(10) $10\dot{2}50\dot{3}23\dot{2}(468$ *Ans.*

$$\begin{array}{r|l}
 & 64 \\ \cline{2-2}
4\times4\times300=4800 & 38503 \\
4\times6\times\ 30=\ \ 720 & \\
6\times6\qquad\quad=\ \ \ \ 36 & \\ \cline{1-1}
5556 & 33336 \\ \hline
46\times46\times300=634800 & 5167232 \\
46\times\ 8\times\ 30=\ \ 11040 & \\
8\times\ 8\qquad\quad=\ \ \ \ \ \ 64 & \\ \cline{1-1}
645904 & 5167232 \\ \cline{2-2}
\end{array}$$

(11) $52\dot{9}47\dot{5}12\dot{9}(809$ *Ans.*

$$\begin{array}{r|l}
 & 512 \\ \cline{2-2}
8\times\ 8\times300=\ \ \ 19200 & 17475 \\ \hline
80\times80\times300=1920000 & 17475129 \\
80\times\ 9\times\ 30=\ \ \ \ 21600 & \\
9\times\ 9\qquad\quad=\ \ \ \ \ \ \ \ 81 & \\ \cline{1-1}
1941681 & 17475129 \\ \cline{2-2}
\end{array}$$

(12)

	958585256(986 *Ans.*
	729
9×9×300=24300	229585
9× 8×30= 2160	
8×8 = 64	
26524	212192
98×98×300=2881200	17393256
98× 6× 30= 17640	
6× 6 = 36	
2898876	17393256

(13)

	14760213677(2453 *Ans.*
	8
2×2×300=1200	6760
2×4× 30= 240	
4×4 = 16	
1456	5824
24×24×300=172800	936213
24× 5× 30= 3600	
5× 5 = 25	
176425	882125
245×245×300=18007500	54088677
245× 3× 30= 22050	
3× 3 = 9	
18029559	54088677

```
                         .  .  .  .
(14)                     128100283921(5041 Ans.
                         125
                         -----
5× 5×300=       7500 |   3100
                -----    ------
50×50×300=    750000 |   3100283
50× 4× 30=      6000 |
 4× 4     =       16 |
              ------ |
              756016 |   3024064
             ------- |   -------
504×504×300=76204800 |    76219921
504×  1× 30=   15120 |
  1×  1    =       1 |
            -------- |
            76219921 |    76219921
                          --------
```

```
                     .  . .
(15)                 53.157376)3.76 Ans.
                     27
                     -----
3×3×300=2700     |26157
3×7× 30= 630     |
7×7     =  49    |
         ----    |
         3379    |23653
         ----    |-----
37×37×300=410700 | 2504376
37× 6× 30=  6660 |
 6× 6     =   36 |
          ------ |
          417396 | 2504376
                   -------
```

```
                       .  .  .
(16)                   .199176704(.584 Ans.
                        125
                        ---
5×5×300=7500      |  74176
5×8× 30=1200      |
8×8     =  64     |
        ----      |
        8764      |  70112
        ----      |  -----
58×58×300=1009200 |   4064704
58× 4× 30=   6960 |
 4× 4     =    16 |
          ------- |
          1016176 |   4064704
                      -------
```

(17) $\sqrt[3]{216}=6.$ $\sqrt[3]{343}=7.$ *Ans.* $\frac{6}{7}$

(18) $\sqrt[3]{2744}=14.$ $\sqrt[3]{6859}=19.$ *Ans.* $\frac{14}{19}$

(19) $\frac{48778}{118638}=\frac{24389}{59319},$ $\sqrt[3]{24389}=29.$ $\sqrt[3]{59319}=39.$ *Ans.* $\frac{29}{39}.$

(20) $5\frac{104}{125}=\frac{729}{125},$ $\sqrt[3]{729}=9$ $\sqrt[3]{125}=5$ *Ans.* $\frac{9}{5}=1\frac{4}{5}.$

(21) $2(1.25992+$ *Ans.*

$$\begin{array}{r|r}
 & 2(1.25992+ \\
 & 1 \\ \hline
1\times1\times300=300 & 1000 \\
1\times2\times\ 30=\ 60 & \\
2\times2\qquad=\ \ \ 4 & \\ \hline
364 & 728 \\ \hline
12\times12\times300=43200 & 272000 \\
12\times\ 5\times\ 30=\ 1800 & \\
5\times\ 5\qquad=\ \ \ 25 & \\ \hline
45025 & 225125 \\ \hline
125\times125\times300=4687500 & 46875000 \\
125\times\ \ 9\times\ 30=\ \ 33750 & \\
9\times\ \ 9\qquad=\ \ \ \ 81 & \\ \hline
4721331 & 42491979 \\ \hline
1259\times1259\times300=475524300 & 4383021000 \\
1259\times\ \ 9\times\ 30=\ \ 339930 & \\
9\times\ \ 9\qquad=\ \ \ \ 81 & \\ \hline
475864311 & 4282778799 \\ \hline
12599\times12599\times300=47620440300 & 100242201000 \\
12599\times\ \ 2\times\ 30=\ \ 755940 & \\
2\times\ \ 2\qquad=\ \ \ \ 4 & \\ \hline
47621196244 & 95242392488 \\ \hline
\end{array}$$

(22) 9(2.08008+ *Ans.*

	8
2× 2×300= 1200	1000
20×20×300=120000	1000000
20× 8× 30= 4800	
8× 8 = 64	
124864	998912
208×208×300=12979200	1088000
2080×2080×300=1297920000	1088000000
20800×20800×300=129792000000	1088000000000
20800× 8× 30= 4992000	
8× 8 = 64	
129796992064	1038375936512

(23) 200(5.84803+ *Ans.*

	125
5×5×300=7500	75000
5×8× 30=1200	
8× 8= 64	
8764	70112
58×58×300=1009200	4888000
58× 4× 30= 6960	
4× 4= 16	
1016176	4064704
584×584×300=102316800	823296000
584× 8× 30= 140160	
8× 8= 64	
102457024	819656192
5848× 5848×300= 10259731200	3639808000
58480×58480×300=1025973120000	3639808000000
58480× 3× 30= 5263200	
3× 3= 9	
1025978383209	3077935149627

(24) $9\frac{1}{6}=\dot{9}.\dot{1}66\dot{6}66+(2.092+$ *Ans.*

$$\begin{array}{r|l}
 & 8 \\ \cline{2-2}
2\times\ 2\times300=\quad 1200 & 1166 \\ \hline
20\times20\times300=120000 & 1166666 \\
20\times\ 9\times\ 30=\quad 5400 & \\
9\times\ \ 9=\quad 81 & \\ \cline{1-1}
125481 & 1129329 \\ \hline
209\times209\times300=13104300 & \ \ 37337666 \\
209\times\ \ 2\times\ 30=\quad 12540 & \\
2\times\ \ 2=\quad 4 & \\ \cline{1-1}
13116844 & \ \ 26233688 \\ \hline
\end{array}$$

(25) $\sqrt[3]{1953.125}=12.5$ ft. *Ans.*

(26) $3\times3\times3=27=$cu. in. in a cube, of which each side is 3 in. $1728\div27=64$ *Ans.*

(27) $2\times2\times2=8=$ cubic inches in the cubical box. $\frac{1}{4}\times\frac{1}{4}\times\frac{1}{4}=\frac{1}{64}=$ solidity of a cubical block, each side of which is $\frac{1}{4}$ of an inch. $8\div\frac{1}{64}=512$ *Ans.*

(28) A solid half foot is a cube, each side of which is 6 in.; it contains $6\times6\times6=216$ cu. in.; half a cubic foot$=\frac{1}{2}$ of $1728=864$ cu. in.; $864-216=648$ cu. in. *Ans.*

(29) $\overset{\text{cu. ft.}}{288}\times\overset{\text{cu. ft.}}{216}\times\overset{\text{ft.}}{48}=2985984$, $\sqrt[3]{2985984}=144$ *Ans.*

(30) $1728\times3=5184$, $\sqrt[3]{5184}=17.306+$ inches *Ans.*

ARTICLE 297.

(31) $6^3=216$; $3^3=27$. As $216 : 27 :: 32$ lb. : 4 lb. *Ans.*

(32) $11\times11\times11=1331$ *Ans.*

ARTICLE 298.

(1) $216=2\times2\times2\times3\times3\times3$: $\sqrt[3]{216}=2\times3=6$ *Ans.*

(2) $\sqrt[3]{(27\times\ 64)}=3\times4=12$ *Ans.*

(3) $\sqrt[3]{(125\times343)}=5\times7=35$ *Ans.*

ARITHMETICAL PROGRESSION.

ARTICLE 301.—CASE I.

(3) $50-1=49$; $49\times3+2=149$ *Ans.*

(4) $100-1=99$; $99\times5+9=504$ cts.$=\$5.04$ *Ans.*

(5) $54-1=53$; $53\times2=106$; $140-106=34$ *Ans.*

(6) $20-1=19$; common difference$=\$216-\$200=\$16$; $\$16\times19+\$216=\$520$ *Ans.*

(7) $99-1=98$; $98\times\frac{7}{8}=85\frac{3}{4}$; $329-85\frac{3}{4}=243\frac{1}{4}$ *Ans.*

ARTICLE 302.—CASE II.

(2) $300-3=297$; $10-1=9$; $297\div9=33$ *Ans.*

(3) $50-5=45$; $10-1=9$; $45\div9=5$ *Ans.*

ARTICLE 303.

(1) $49-5=44$; $44\div4=11$; $11+1=12$ *Ans.*

ARTICLE 304.—CASE III.

(2) $50+2=52$; $52\times24=1248$; $1248\div2=624$ *Ans.*

(3) $1+12=13$; $13\times12=156$; $156\div2=78$ *Ans.*

(4) $10000-1=9999$; $9999\times1+1=10000$, last term; $1+10000=10001$; $10001\times10000=100010000$; $100010000\div2=50005000$ *Ans.*

(5) The number of terms is evidently 100. The boy travels 6 yards to put the first apple in the basket, 12 the second, and so on; hence, the first term is 6, and the common difference 6.

$100-1=99$; $99\times6+6=600$, last term.
$6+600=606$; $606\times100=60600$; $60600\div2=30300$ yd.
30300 yd.$=17$ mi. 380 yd. *Ans.*

(6) $30-1=29$; $29\times\frac{1}{4}=7\frac{1}{4}$; $30-7\frac{1}{4}=22\frac{3}{4}$ mi. distance traveled last day. $30+22\frac{3}{4}=52\frac{3}{4}$; $52\frac{3}{4}\times30=1582\frac{1}{2}$ mi.; $1582\frac{1}{2}\div2=791\frac{1}{4}$ mi. *Ans.*

(7) Common difference$=193\times2=386$ in.
$60-1=59$; $386\times59+193=22967$ inches, distance fallen in the last second.
$193+22967=23160$; $23160\times60=1389600$; $1389600\div2=694800$ in.$=57900$ ft. *Ans.*

GEOMETRICAL PROGRESSION.

ARTICLE 307.—CASE I.

(3) $2^{12}=4096$; $4096\times2=8192$ *Ans.*

(4) $4^{8}=65536$; $262144\div65536=4$ *Ans.*

(5) Ratio$=3$; $3^{9}=19683$; $19683\times10=196830$ *Ans.*

(6) $2^{34}=17179869184$; $17179869184\times1=17179869184$ *Ans.*

(7) $3^{34}=16677181699666569$, which, multiplied by 1, gives the same number for the 35th term.

ARTICLE 308.—CASE II.

(2) $3^{6}=729$; $729\times10=7290$, last term. $7290\times3=21870$; $21870-10=21860$; $21860\div2=10930$ *Ans.*

(3) $2^{11}=2048$, $2048\times1=2048=$last term. $2048\times2=4096$, $4096-1=4095$, and $4095\div1=4095$ *Ans.*

(4) $4^{11}=4194304$, $4194304\times1=4194304$, last term.
$4194304\times4=16777216$, $16777216-1=16777215$,
$4-1=3$, $16777215\div3=5592405$ cts.$=\$55924.05$ *Ans.*

(5) Ratio$=2$, $2^{39}=549755813888$, this multiplied by 10, gives 5497558138880, for the last term.
$5497558138880\times2=10995116277760$; from this subtract 10, and divide by $2-1=1$, and we get for the
Ans. \$109951162777.50

(6) Ratio=3, 3^{31}=617673396283947,
617673396283947×3=1853020188851841
1
3—1=2)1853020188851840
$9265100944259.20 *Ans.*

(7) .3×10=3 ; 10—1=9 ; 3÷9=$\frac{1}{3}$ *Ans.*

(8) 100×1.04=104 ; 1.04—1=.04, 104÷.04=2600 *Ans.*

(9) Ratio=3, $\frac{1}{3}$×3=1 ; 3—1=2 ; 1÷2=$\frac{1}{2}$ *Ans.*

(10) Ratio=2, $\frac{1}{2}$×2=1 ; 2—1=1 ; 1÷1=1 *Ans.*

Article 309.—Permutation.

(1) 1×2×3×4×5=120 *Ans*

(2) 1×2×3×4×5×6×7×8×9×10×11×12
=479001600 *Ans.*

(3) 1×2×3×4×5×6×7×8=40320 days
=110 yr. 170 da. *Ans.*

(4) 1×2×3×4×5×6×7×8×9×10×11×12×13×14×15
16×17×18×19×20×21×22×23×24×25×26
=403291461126605635584000000 *Ans.*

The subject of permutations and combinations properly belongs to Algebra. (See Ray's Algebra, Part 2d.)

Article 311.—Mensuration.

(1) 17×15=255 sq. ft. *Ans.*

(2) 2 ft. 3 in.=2.25 ft., 12 ft. 6 in.=12.5 ft., 2.25×12.5
=28.125=28$\frac{1}{8}$ sq. ft. *Ans.*

(3) 15 in.=1$\frac{1}{4}$ ft. ; 16 ft.×1$\frac{1}{4}$ ft.=20 sq. ft. *Ans.*

(4)

	ft.	in.
	12	6
	1	2
	12	6
	2	1
Ans.	14	7

Ans. $14\frac{7}{12}$ sq. ft.=14 sq. ft. 84 sq. in.

Or, 1 ft. 2 in.=14 in., 12 ft. 6 in.=150 in.
150×14=2100 sq. in.=14 sq. ft. 84 sq. in.

(5)

ft.	in.	
5	7	
1	10	
5	7	
4	7	10
10	2	10

	$ cts.
	1.50
	10
	15.00
2 in.=$\frac{1}{6}$	.25
6″ =$\frac{1}{4}$	.0625
4″ =$\frac{1}{6}$	.0416+
Ans.	$15.354+

(6) 120 rd.× 84 rd.=10080 P.=63 A. *Ans.*

(7) 65 rd.× 65 rd.= 4225 P.=26 A. 1 R. 25 P. *Ans.*

(8) 35 rd.× 16 rd.= 560 P.= 3 A. 2 R. *Ans.*

(9) 693×693=480249 sq. ft.=1764 P.=11 A. 4 P. *Ans.*

(10) 15× 15=225, 225+225=450 sq. ft., 30×30=900 sq. ft.
900—450=450 sq. ft. *Ans.*

(11) 80÷ 10=8 ft. *Ans.*

(12) 10×144=1440 sq. in., 1440÷10=144 in.=12 ft. *Ans.*

(13) 6× 5=30 sq. yd., 30÷$1\frac{1}{2}$=20 yd. *Ans.*

(14) 21 ft. 3 in.=21.25 ft., 13 ft. 6 in.=13.5 ft., 21.25×13.5=286.875 sq. ft., $1\frac{1}{4}$ yd.=$3\frac{3}{4}$=3.75 ft., 286.875÷3.75=76.5 ft.=25.5 yd.=$25\frac{1}{2}$ yd.

(15) 3×$1\frac{1}{2}$=$4\frac{1}{2}$ sq. yd., $4\frac{1}{2}$÷$\frac{3}{4}$=$\frac{9}{2}$×$\frac{4}{3}$=6 yd. *Ans.*

ARTICLE 312.

(1) ft. in.
25 9
21 3
540 9
6 5 3
9)547 2 3
Ans 60 sq. yd. 7 sq. ft.+

(2) ft. in.
22 7
13 11
293 7
20 8 5
9)314 3 5
34 8 3 5
.20
6.80
.17777+ =cost 8sq. ft.
.00555+ = " 3′
.00077+ = " 5″
Ans. $6.9841+

(3) ft. in.
20 6
20 6
16 3
16 3
73 6=circumference of room.
10 4
735 0
24 6
759 6=walls.
333 1 6
1092 7 6
94 2 6=deductions.
9)998 5

110 sq. yd. 8 sq. ft. and 5′=$\frac{5}{12}$ sq. ft. [*Ans.*

ft. in.
20 6
16 3
328 0
5 1 6
333 1 6=ceiling.

Deductions.
26 0 6
29 2
39 0
94 2 6

ft. in.
6 3
4 2
25 0
1 0 6
26 0 6=fire-place.

ft. in.
4 2
7
29 2=door.

ft. in.
3 3
6
19 6
2
39 0=windows.

(4)
ft. in.
20 0
20 0
14 6
14 6
69 0=circumference [of room.
10 4
690 0
23
713 0
55 4=deductions.
9)657 8
73 sq. yd. 8′
27
511
146
19.71
8′=$\frac{2}{27}$ 2
Ans. $19.73

ft. in.
4 4
4
17 4=fireplace.

ft. in.
3 2
6
19
19
17 4
55 4

(5)
ft. in.
35 4
8 3
282 8
8 10
9)291 6
32 sq. yd. 3 sq. ft. [6′
18
256
32
576
3 sq. ft.=$\frac{1}{3}$ 6
6′ =$\frac{1}{6}$ 1
Ans. $5.83

(6)
yd.
21
15
105
21
315 =sq. yd. in whole.
$36\frac{3}{4}$=sq. yd. in foot-path.
$278\frac{1}{4}$
24
1112
556
6
$66.78

5 ft. 3 in.=$5\frac{1}{4}$ ft.=$1\frac{3}{4}$ yd.

$21\times1\frac{3}{4}=36\frac{3}{4}$ sq. yd.

$36\frac{3}{4}$
36
216
108
27
13.23
66.78
$80.01 *Ans.*

(7)

```
ft.  in.  ft.
75   6=75.5
12   6=12.5
     ------
       3775
      1510
      755
     ------
  9)943.75=sq. ft.
    ------
    104.86+ sq. yd.
        10
    ------
    1048.6+ cts.
Ans. $10.486+
```

(8)

```
ft.  in.
 6   11
 5    4
 4    3
--------
16    6
 3    6
--------
49    6
 8    3
--------
57    9
  7=windows in a line.
--------
404   3
 16 cts.
    |2424
    |404
3′=¼|   4
    ------
Ans. $64.68
```

(9)

```
ft. in.  ft.
36  3=36.25
16  6= 16.5
       -----
      18125
     21750
     3625
     -------
1|00)5|98.125
     --------
     5.98125
           3
     -------
Ans. $17.94375
```

(10)

```
ft. in.
 4   6
 6
-------
27   0=fire-place.

ft. in.
10   6
 8
-------
84   0=stairs.
27
-------
111  0
```

```
      ft.
      35
      30
     ----
     1050=sq. ft.
      111=deductions.
     ----
1|00)9|39
     ----
     9.39=squares.
       $5
     ----
Ans. $46.95
```

(11)

```
     ft. in.
     18   6
     18   6
     -------
     37   0
     40
     ----
1|00)14|80=sq. ft.
     -----
     14.80=squares.
     $3.50
     -----
     74000
    4440
    ------
Ans. $51.8000
```

ARTICLE 313.

(1) $12\times15\div2=90$ sq. ft. *Ans.*

(2) $44\times18\div2=396$ P.$=2$ A. 1 R. 36 P. *Ans.*

(3) $80\times67\div2=2680$ P.$=16$ A. 3 R. *Ans.*

(4) 12 ft. 6 in.$=12.5$ ft., 16 ft. 9 in.$=16.75$ in., $12.5\times16.75\div2=104.6875$ sq. ft., 104.6875×12 $=1256.25$ cts.$=\$12.56\frac{1}{4}$ *Ans.*

(5) $13+14+15=42$, $42\div2=21$, $21-13=8$, $21-14=7$, $21-15=6$, $21\times8\times7\times6=7056$, $\sqrt{7056}=84$ sq. ft. *Ans.*

(6) $2+3+4=9$, $9\div2=4.5$; $4.5-2=2.5$; $4.5-3=1.5$; $4.5-4=.5$; $4.5\times2.5\times1.5\times.5=8.4375$; $\sqrt{8.4375}$ $=2.9047375+$ sq. ft. *Ans.*

ARTICLE 314.

(1) $35+26=61$; $61\times11=671$; $671\div2=335\frac{1}{2}$ sq. in. *Ans.*

(2) $19+25=44$; $44\times32=1408$; $1408\div2=704$ P. $=4$ A. 1 R. 24 P. *Ans.*

ARTICLE 315.

(1) $48\times3.1416=150.7968$ ft. *Ans.*

(2) $4\times3.1416=12.5664$ ft.$=12$ ft. 6.7968 in. *Ans.*

(3) $7912.4\times3.1416=24857.59584$ mi. *Ans.*

ARTICLE 316.

(1) $15\div3.1416=4.7746+$ft.$=4$ ft. $9.295+$in. *Ans.*

(2) 12 ft. 5 in.$=12.4166+$; $12.4166\div3.1416=3.952338$ ft. $=3$ ft. $11.428+$in. *Ans.*

ARTICLE 317.

(1) $42\div2=21$; $21\times21=441$; 3.1416×441 $=1385.4456$ sq. ft. *Ans*

(2) $7\frac{1}{2}$ rods$=7.5$; $7.5\times7.5=56.25$; 3.1416×56.25 $=176.715$ P.$=1$ A. 16.715 P. *Ans.*

ARTICLE 318.

(1) 962.115÷.7854=1225 ; $\sqrt{1225}=35$=diameter ; 35×3.1416=109.956 *Ans.*

(2) 160÷.7854=203.71785077+ ; $\sqrt{203.71785077}$ =14.2729+ ; 14.2729÷2=7.1364+ rods ; or 117 ft. 9+in. *Ans.*

ARTICLE 320.

(1) 3 ft. 3 in.=$3\frac{1}{4}$ ft. ; 4 ft. 4 in.=$4\frac{1}{3}$ ft. ; $12\times3\frac{1}{4}\times4\frac{1}{3}$ =169 cu. ft. *Ans.*

(2) 2 ft. 6 in.=$2\frac{1}{2}$ ft. ; 1 ft. 9 in.=$1\frac{3}{4}$ ft. ; $6\times2\frac{1}{2}\times1\frac{3}{4}$ =$26\frac{1}{4}$ cu. ft. *Ans.*

(3) 3 ft. 2 in.=$3\frac{1}{6}$; 2 ft. 8 in.=$2\frac{2}{3}$; 2 ft. 6 in.=$2\frac{1}{2}$ ft.
$3\frac{1}{6}\times2\frac{2}{3}\times2\frac{1}{2}=\frac{19}{6}\times\frac{8}{3}\times\frac{5}{2}=\frac{190}{9}=21\frac{1}{9}$ cu. ft.
$21\frac{1}{9}\times81$ cts.=$17.10 cts. *Ans.*

(4)	ft.	in.		
	4	10		
	2	11		
	9	8		
	4	5	2	
	14	1	2	
	2	2		
	28	2	4″	
	2	4	2	4‴
Ans.	30	6′	6″	4‴

ARTICLE 321.

(1) 97 ft. 5 in.=97.416+ft. ; 18 ft. 3 in.=18.25 ft. ; 2 ft. 3 in. =2.25 ft. ; 97.416×18.25×2.25=4000.1445 cu. ft.
4000.1445÷24.75=161.6+P. *Ans.*

(2) 53 ft. 6 in.=53.5 ft. ; 12 ft. 3 in.=12.25 ft., 53.5 ×12.25×2=1310.75 cu. ft.=52.95+P. *Ans.*

(3) 53 ft. 6 in.=53.5 ft.; 12 ft. 6 in.=12.5 ft.; 53.5×12.5×2 =1337.5 cu. ft.=54.0404+ P., 54.0404×$2.25 =$121.59+ *Ans.*

(4) $48\frac{1}{3}\times16\frac{1}{2}\times1\frac{1}{2}=\frac{145}{3}\times\frac{33}{2}\times\frac{3}{2}=\frac{14355}{12}=1196\frac{1}{4}$ cu. ft. $1196\frac{1}{4}\times20=23925$ bricks *Ans.*

(5) 15 in.=$1\frac{1}{4}$ ft.: $320\times6\times1\frac{1}{4}=2400$ cu. ft.; 2400×20 =48000 bricks; 48000×$5.875÷1000=$282 *Ans.*

(6) $120\times8\times1\frac{1}{2}=1440$ cu. ft.=2488320 cu. in. in wall. 8×4×2.25=72 cu. in. in each brick; 2488320÷72 =34560 bricks *Ans.*

(7) 240×6×3=4320 cu. ft.=7464960 cu. in. in wall. 9×4×2=72 cu. in. in each brick, 7464960÷72=103680 bricks. 103680÷1000=103.68; $3.25×103.68 =$336.96 *Ans.*

ARTICLE 322.

(1) 2+2+2=6; 6÷2=3; 3—2=1; 3—2=1; 3—2=1. 3×1×1×1=3; $\sqrt{3}$=area of base. $14\times\sqrt{3}=14\sqrt{3}$ =area of base. $14\sqrt{3}=\sqrt{196}\times\sqrt{3}=\sqrt{588}$ =24.2487+cu. in. *Ans.*

(2) 4×4=16; .7854×16=12.5664=area of end. 12.5664×12=150.7968 cu. ft. *Ans.*

(3) $18\frac{1}{2}=18.5$: $18.5^2=342.25$: 342.25×.7854=268.80315 cu. in.=area of end. 268.80315×8=2150.4252 cu. in. *Ans.*

(4) $6^2\times.7854=28.2744$=area of end. 28.2744×8 =226.1952 cu. ft.=390865.3056 cu. in. 390865.3056 ÷2150.4=181.764 bu. *Ans.*

(5) 15×5×4=300 cu. ft.=518400 cu. in.; 518400÷2150.4 =241+bu. *Ans.*

(6) 12×3×5=180 cu. ft.=311040 cu. in.; 311040÷2150.4 =144.6+bu. *Ans.*

ARTICLE 323.

(1) $5\times5=25$ sq. ft.=area of base. $25\times21\div3=175$ cu. ft. *Ans.*

(2) $10^2\times.7854=78.54$, area of base. $78.54\times15\div3=392.7$ cu. ft. *Ans.*

(3) 37 ft. 8 in.=452 in., $452^2\times.7854=160460.3616$=area of base. 79 ft. 9 in.=957 in. $160460.3616\times957\div3=51186855.3504$ cu. in.=29622.0227+cu. ft. *Ans.*

(4) $720\times720=518400$=area of base. $518400\times477\div3=82425600$ cu. ft.=3052800 cu. yd. *Ans.*

(5) $1\frac{1}{2}=1.5$: $1.5^2\times.7854=1.76715$=area of base. $1.76715\times9\div3=5.30145$=cu. in. in cup. $231\div5.30145=43.57+$times *Ans.*

ARTICLE 324.

(1) $3\times3=9$=area of lower base. $2\times2=4$=area of upper base. $9\times4=36$; $\sqrt{36}=6$=area of mean base. $9+4+6=19$; $19\times\frac{12}{3}=76$ cu. ft. *Ans.*

(2) $27\times27=729$=area of lower base. $16\times16=256$=area of upper base. $729\times256=186624$; $\sqrt{186624}=432$=area of mean base. $729+256+432=1417$; $1417\times\frac{224}{3}=105802.6666+$cu. in.=61.2283950+cu. ft. *Ans.*

(3) $2\frac{1}{2}=2.5$: $2.5^2\times.7854=4.90875$=area of mouth. $1^2\times.7854=.7854$=area of bottom. $4.90875\times.7854=3.85533225$; $\sqrt{3.85533225}=1.9635$=area of mean base. $4.90875+.7854+1.9635=7.65765$; $7.65765\times\frac{5}{3}=12.76275$ cu. in. *Ans.*

ARTICLE 325.

(1) $13^3=2197$; $2197\times.5236=1150.3492$ cu. in. $15^3=3375$; $3375\times.5236=1767.15$ cu. in. $30^3=27000$; $27000\times.5236=14137.2$ cu. in. *Ans.*

ARTICLE 326.

(1) 5 ft. 4 in.$\times 3=16$ ft.=circumference of base. 7 ft. 6 in. $=7\frac{1}{2}$ ft., $16\times 7\frac{1}{2}\div 2=60$ sq. ft. *Ans.*

(2) $8\frac{1}{2}\times 3.1416=26.7036$=circumference of base. $26.7036\times 25\div 2=333.795$ sq. ft. *Ans.*

(3) 2 ft. 11 in.=35 in.; $35\times 3.1416=109.956$=circ. of base. $109.956\times 55\div 2=3023.79$ sq. in.=area of conv. surface. $35^2=1225$: $1225\times .7854=962.115$ sq. in.=area of base. $3023.79+962.115=3985.905$ sq. in.$=27.679895+$ sq. ft. *Ans.*

(4) $7912^2=62599744$; 62599744×3.1416 $=196663355.7504$ sq. mi. *Ans.*

ARTICLE 327.

(1) 10 ft.=120 in., 5 ft.=60 in., 4 ft.=48 in. $120\times 60\times 48$ $=345600$ cu. in., $345600\div 231=1496+$gal. *Ans.*

(2) 12 ft.=144 in., 6 ft.=72 in., 10 ft.=120 in., $144\times 72\times 120=1244160$ sq. in., $1244160\div 2150.4$ $=578.57+$bu. *Ans.*

ARTICLE 328.

(1) 4 ft.=48 in.; 6 ft.=72 in.; $48^2=2304$; 2304×72 $=165888$; $165888\times .7854=130288.4352$ cu. in.$=564.019$ $+$gal.$=17.9+$bl. *Ans.*

(2) 6 ft.=72 in.; 9 ft.=108 in.; $72^2=5184$; $5184\times 108=$ 559872; $559872\times .7854=439723.4688$ cu. in. $=1903.564+$gal.$=60.43+$bl. *Ans.*

ARTICLE 329.

(1) $40\times 30=1200$; $40-30=10$; $10\times 10=100$; $100\div 3=33\frac{1}{3}$; $1200+33\frac{1}{3}=1233\frac{1}{3}$; $1233\frac{1}{3}\times 50=61666\frac{2}{3}$; $61666\frac{2}{3}\times .7854$ $=48433$ cu. in.$=209\frac{2}{3}=209.66+$gal. *Ans.*

(2) 7 ft. 6 in.=90 in.; 10 ft.=120 in.; 12 ft. 6 in.=150 in.; $90\times120=10800$; $120-90=30$; $30^2=900$; $900\div3=300$; $10800+300=11100$; $11100\times150=1665000$; $1665000\times.7854=1307691$ cu. in.=5661 gal. $=179\frac{5}{7}$ bl. *Ans.*

ARTICLE 330^a.

(1) $40-31=9$; $\frac{2}{3}$ of $9=6$; $31+6=37$; $37\times37=1369$; $1369\times50\times.0034=232.73$ gal. *Ans.*

(2) $32-30=2$; $2\times\frac{6}{10}=1.2$: $30+1.2=31.2$; $31.2\times31.2=973.44$; $973.44\times40\times.0034=132.38+$gal. *Ans.*

ARTICLE 330^b.

(1) 1 A.=160 P. : $160\div15=10\frac{2}{3}$ rd. *Ans.*

(2) $24\times18\frac{1}{2}\times10\frac{7}{12}=4699$ cu. ft. *Ans.*

(3) $144\div.7854=183.34606569$; $\sqrt{183.34606569}=13.5405+$in. *Ans.*

(4) $1728\div.5236=3300.229182+$; $\sqrt[3]{3300.229182}=14.8884+$in. *Ans.*

(5) $30+40+50=120$; $120\div2=60$; $60-30=30$; $60-40=20$; $60-50=10$. $60\times30\times20\times10=360000$; $\sqrt{360000}=600$ sq. ft.$=66\frac{2}{3}$ sq. yd. *Ans.*

(6) 12 ft. 6 in.=150 in.; $15+11=26$; $26\times150\div2=1950$ sq. in.$=13\frac{13}{24}$ sq. ft. *Ans.*

(7) $16^2\times.7854=201.0624$, area of greater circle. $10^2\times.7854=78.54$, area of inner circle. $201.0624-78.54=122.5224$ sq. ft. *Ans.*

(8) $6+6+2+2=16$=circumference. $16\times2\frac{1}{2}=40$=area of sides. $6\times2=12$=area of bottom. $12+40=52$=sq. ft. of sheet lead. $52\times8=416$ lb. of lead. 4 cts.$\times416=\$16.64$ *Ans.*

(9) 5 ft.=60 in.; 4 ft.=48 in.; $60\times48\times48=138240$ s. in.; $138240\div2150.4=64.28+$bu. 25 cts.$\times64.28=\$16.07$ *Ans.*

(10) $1448 \div 3.1416 = 460.91163+$ ft.=diameter ; $460.91163+ \times 1448 = 667400+$; $667400+ \div 4 = 166850+$ sq. ft.=3 A. 3 R. 12 P. 25 sq. yd. 8+sq. ft. *Ans.*

(11) $37^2 = 1369$ sq. in.= surface of one side : $1369 \times 6 = 8214$ sq. in. *Ans.*

ARTICLE 332.

(1) $1236 \div 3 = 412$ *Ans.*

(2) $675 + 225 = 900$: $900 \div 2 = 450$ *Ans.*

(3) $292 \times 3 = 876$: $876 \div 4 = 219$ *Ans.*

(4) 5 lb. 15 oz.=$5\frac{15}{16}$ lb. : $\$1.20 \times 5\frac{15}{16} = \$7.12\frac{1}{2}$ *Ans.*

(5) 13 bu. 3 pk.=$13\frac{3}{4}$ bu. : $\$1.45 \times 13\frac{3}{4} = \$19.93\frac{3}{4}$ *Ans.*

(6) $\$1000 + \$600 = \$1600$: $\frac{1000}{1600} = \frac{5}{8}$; $\frac{600}{1600} = \frac{3}{8}$; $\frac{5}{8}$ of $320 = 200$ A.=A's share : $\frac{3}{8}$ of 320 A.=120 A.=B's share. *Ans.*

(7) $42\frac{1}{2} \div 3\frac{3}{4} = \frac{85}{2} \times \frac{4}{15} = \frac{17 \times 2}{3} = \frac{34}{3}$ hr.=11 hr. 20 min. *Ans.*

(8) $14\frac{3}{4} \div 1\frac{3}{8} = \frac{59}{4} \times \frac{8}{11} = \frac{59 \times 2}{11} = \frac{118}{11} = 10\frac{8}{11}$. *Ans.*

(9) 6 times—3 times the number=3 times the number=81 : $81 \div 3 = 27$ *Ans.*

(10) $\$\frac{3}{5} \times 4 = \$\frac{12}{5}$: $\$\frac{7}{8} \times 10 = \$\frac{70}{8} = \frac{35}{4}$: $\$\frac{12}{5} + \frac{35}{4} = \frac{48}{20} + \frac{175}{20} = \$\frac{223}{20}$: $\$\frac{223}{20} \div \$\frac{1}{10} = \frac{223 \times 10}{20} = \frac{223}{2} = 111\frac{1}{2}$ yd. *Ans.*

(11) $1 - \frac{2}{3} = \frac{1}{3}$: $\frac{1}{4}$ of $\frac{1}{3} = \frac{1}{12}$: $\frac{1}{3} - \frac{1}{12} = \frac{1}{4} = \$125$: $\$125 \times 4 = \$500$ *Ans.*

(12) $2\frac{7}{16} + 1\frac{5}{8} = 2\frac{7}{16} + 1\frac{10}{16} = 4\frac{1}{16}$: $2\frac{7}{16} - 1\frac{10}{16} = \frac{13}{16}$: $4\frac{1}{16} = \frac{65}{16}$: $\frac{65}{16} \times \frac{13}{16} = \frac{845}{256} = 3.30078125$ *Ans.*

(13) $60 - 19 = 41$ yr.= the time since marriage : $41 + 21 = 62$ yr. *Ans.*

(14)

$$\begin{array}{r|rrrrr} 2 & 8 & 12 & 21 & 36 & 48 \\ \hline 2 & 4 & 6 & 21 & 18 & 24 \\ \hline 2 & 2 & 3 & 21 & 9 & 12 \\ \hline 3 & 1 & 3 & 21 & 9 & 6 \\ \hline & 1 & 1 & 7 & 3 & 2 \end{array}$$

$$\begin{array}{l} 65)143(2 \\ \underline{\;\;\;\;130} \\ \;\;\;13)65(5 \\ \;\;\;\;\;\;\;\underline{65} \end{array}$$

13=greatest common divisor.

$2 \times 3 \times 7 \times 3 \times 2 \times 2 \times 2 = 1008$, least common multiple.

$\frac{1008}{13} = 77\frac{7}{13}$ *Ans.*

(15) 3 mi. 5 fur. 110 yd.=233640 in. : 233640 in.÷39.371 in. =5934.317+meters *Ans.*

(16) 800000000÷250=3200000 min.=14 yr. 223 da. 3 hr. 20 min. *Ans.*

(17) $16\frac{2}{3}=\frac{50}{3}$. $12.625\div\frac{50}{3}=\frac{12.625\times3}{50}=.7575$ *Ans.*

(18) yr. mon. da.
1855 5 10
1854 5 20
11 20=$11\frac{2}{3}$ mon.=$\frac{35}{36}$ yr. : $\$250\times\frac{35}{36}=\$243\frac{1}{18}$ *Ans.*

(19) 1 lb. of each costs 12+30+5=47 cts. : \$47÷47 cts.= 100 lb. *Ans.*

(20) $\frac{2}{3}-\frac{3}{8}=\frac{16}{24}-\frac{9}{24}=\frac{7}{24}$. If $\frac{7}{24}$=35 gallons, $\frac{1}{7}$ of 35 gallons, which is 5 gallons, will be $\frac{1}{24}$, and the cistern, when full, will contain 5×24=120 gallons.

(21) 60×5=\$300 : \$4×23=\$92 : 60—23=37 bl. \$300+\$51 =\$351 ; \$351—\$92=\$259 ; \$259÷37=\$7 *Ans.*

(22) 12 cwt. 1 qr. 7 lb.=1232 lb. : 3 qr. 13 lb.=88 lb. : 1232÷88=14 *Ans.*

(23) $10^2\times3.1416$=314.16 sq. in. 5cts.×314.16=\$15.708 *Ans.*

(24) If 1 ox is worth 8 sheep, 3 oxen will be worth 24 sheep, but 2 horses are worth 3 oxen, therefore 2 horses are worth 24 sheep; hence, 1 horse is worth 12 sheep, and at \$2.50 a piece, 12 sheep are worth \$30 *Ans.*

(25) If $\frac{2}{3}$ of \$1, buy $\frac{1}{5}$ of a sheep, 1 sheep is worth 5 times $\$\frac{2}{3}=\frac{10}{3}$, and $\frac{3}{7}$ of a sheep is worth $\frac{3}{7}$ of $\$\frac{10}{3}=\$\frac{10}{7}$; hence, $\frac{1}{14}$ of an ox is worth $\$\frac{10}{7}$, and 1 ox is worth 14 times $\$\frac{10}{7}$ =\$20; and 10 oxen are worth 10 times \$20=\$200 *Ans.*

(26) 19 is $2\frac{1}{9}$ times 9, therefore the required number must be $2\frac{1}{9}$ times 54 : $54\times2\frac{1}{9}=114$ *Ans.*

(27) $\frac{1}{2}:\frac{3}{4}=\frac{3}{4}\div\frac{1}{2}=\frac{3}{2}$; $\frac{2}{3}$ of $\frac{3}{2}=1$; 1 is 3 times $\frac{1}{3}$: $\frac{1}{3}\times3=1$ *Ans.*

(28) $13\times5\frac{1}{4}=68\frac{1}{4}$ hr. required to perform the work. $68\frac{1}{4}\div9 =\frac{273}{4}\times\frac{1}{9}=\frac{91}{12}=7\frac{7}{12}$ days of 9 hr. each.

(29) \$40 : \$100 : : \$15 : \$37.50 *Ans.*

(30) From Monday noon till 6 P. M. of Thursday is 78 hr.

which is $3\frac{1}{4}$ days of 24 hr. each; hence the clock has gained $3\frac{1}{4}\times7\frac{1}{2}$ min. $7\frac{1}{2}\times3\frac{1}{4}=\frac{15}{2}\times\frac{13}{4}=\frac{195}{8}=24\frac{3}{8}$ min., therefore the time by the clock will be 6 hrs. $24\frac{3}{8}$ min.

(31) If 7 men can mow 35 A. in 4 days, 1 man can mow $\frac{1}{7}$ of 35 A., which is 5 A, in 4 days; and in 1 day 1 man can mow $\frac{1}{4}$ of 5 A., which is $1\frac{1}{4}$ A.; if 1 man can mow $1\frac{1}{4}$ A. in 1 day, 10 men can mow 10 times $1\frac{1}{4}$ A., which is $12\frac{1}{2}$ A, in 1 day, and in $3\frac{1}{2}$ days they can mow $3\frac{1}{2}$ times $12\frac{1}{2}$ A.$=\frac{25}{2}\times\frac{7}{2}=\frac{175}{4}=43\frac{3}{4}$ A. *Ans.*

(32) $35\frac{7}{8}-16\frac{1}{4}=19\frac{5}{8}$ yd.: $\$1\frac{1}{4}\times16\frac{1}{4}=\$20\frac{5}{16}$:
$\$\ \frac{3}{4}\times35\frac{7}{8}=\$26\frac{29}{32}$: $\$\ \frac{5}{8}\times19\frac{5}{8}=\$12\frac{17}{64}$:
$\$20\frac{5}{16}+\$12\frac{17}{64}-\$26\frac{29}{32}=\$5\frac{43}{64}=\$5.67+$ *Ans.*

(33) 11 hr.$\times6=66$ hr. : $10\times66=660$ cu. ft.
660 cu. ft.$\div(5\times2)=66$ ft. *Ans.*

(34) If 4 men can do the work in 20 days, 1 man can do it in 4 times 20 days=80 days; hence 1 man can do $\frac{1}{80}$, and 3 men $\frac{3}{80}$ of it in one day. If 6 women can do the work in 20 days, 1 woman can do it in 6 times 20 days=120 days; hence 1 woman can do $\frac{1}{120}$, and 5 women $\frac{5}{120}$ of it in 1 day. Therefore 3 men and 5 women, in 1 day can do $\frac{3}{80}+\frac{5}{120}=\frac{19}{240}$ of the work, and therefore they can do it in $1\div\frac{19}{240}=\frac{240}{19}=12\frac{12}{19}$ days.

(35) $40+60=100$ yd. : 5 yd. : 40 : : \$6 : \$48 :
6 yd. : 60 yd. : : \$9 : \$90 :
5 yd. : 100 yd. : : \$7 : \$140 : \$90+\$48=\$138
\$140—\$138=\$2 *Ans.*

(36) After the first drawing $\frac{40}{50}$ or $\frac{4}{5}$ are pure wine, and when 10 gal. more are drawn, $\frac{4}{5}$ of the remaining 40 gal. are pure wine; $\frac{4}{5}$ of 40 gal.=32 gal. *Ans.*

(37) In 14 days of 10 hr. each, there are 140 hours. If 27 men can do the work in 140 hours, 1 man can do it in 27 times 140 hr.=3780 hr.; and since the work of a boy is half that of a man, it will require a boy twice 3780 hr.=7560 hr. to do the work. If it requires 1 boy 7560 hr. to perform the work, it will take 24 boys $\frac{1}{24}$ of 7560 hr.=315 hr. to do it. 315 hr. ÷45=7 hr. *Ans.*

(38) 16′ 40″×9=2° 30′ : 2° 30′÷15 gives 10 min.
Ans. 10 min. before 9.

(39) $\cancel{2}\cancel{0}$ men : $\cancel{1}\cancel{0}$ men
$\cancel{2}$ $\cancel{2}\cancel{0}.\cancel{4}$ ft. l. : $\cancel{4}\cancel{0}$ ft. l.
$\cancel{3}$ $\cancel{3}.\cancel{3}$ ft. w. : $\cancel{4}.\cancel{5}$ ft. w.
$\cancel{1}.\cancel{5}$ ft. d. : $\cancel{1}.\cancel{1}$ ft. d. [9×1÷1=9 da. *Ans.*
$\cancel{1}\cancel{0}$ hr. : 9 hr. : : $\cancel{1}\cancel{0}.\cancel{2}$ da.

(40) \$1+\$.20=\$1.20 : \$14÷\$1.20=\$11$\frac{2}{5}$ *Ans.*

(41) 9 cows=3 times 3; if 3 cows=2 horses, 9 cows=3 times 2=6 horses: 4+6=10 : $\frac{4}{10}=\frac{2}{5}$: $\frac{6}{10}=\frac{3}{5}$: $\frac{2}{5}$ of \$45=\$18, A's share : $\frac{3}{5}$ of \$45=\$27, B's share.

(42) 35 mi.÷2$\frac{1}{2}$=14 mi. the sum of the rates : 14—5=9 : 9÷2=4$\frac{1}{2}$ mi. one : 4$\frac{1}{2}$+5=9$\frac{1}{2}$ mi. the other.

(43) \$1—\$.17$\frac{1}{2}$=\$.825 : \$14.85÷\$.825=\$18, purchase price. \$18×.12$\frac{1}{2}$=\$2.25 : \$18+\$2.25=\$20.25 *Ans.*

qt. bu. qt.
(44) 1 bu.=32 qt. 32—4=28 qt. 28 : 2 : : 32 :
2 bu. 1 pk. 1$\frac{1}{7}$ qt. *Ans.*

(45) The provisions of 32 men for 5 mon., will last 1 man 160 mon.: the provisions of 1 man for 160 mon., in 8 mon. would be consumed by as many men, as 8 mon. is contained times in 160 mon., 160÷8=20 men, hence 32—20=12, the number of men that must leave.

(46) Int. for 63 da. at 6 %=\$.0105 : \$1—\$.0105=\$.9895 : \$1009.29÷\$.9895=\$1020, the face of the note.
Int. for 60 da. at 12 %=\$.02 : \$1+\$.02=\$1.02 :
\$1020÷\$1.02=\$1000 *Ans.*

(47) \$12÷20=\$.60 gain per yard. The difference between 5 % less, and 10 % more is 15 % : \$.60÷.15=\$4 *Ans.*

(48) 9×9×$\frac{1}{2}$=40$\frac{1}{2}$ cu. in. 14×11×$\frac{5}{8}$=96$\frac{1}{4}$ cu. in. :
40$\frac{1}{2}$: 96$\frac{1}{4}$: : \$3.24 : \$7.70 *Ans.*

(49) 120+60=180 : 180 : 120 : : 3 mon. : 2 mon. *Ans.*

(50) 25—20=5 rd. gain in 25 rd. : 5 rd. : 47 rd. : : 25 rd. :
235 rd. *Ans.*

(51) Int. of \$1 for 4 mon. 3 da.=\$.0205 : \$1—\$.0205 =\$.9795 : \$300÷\$.9795=\$306.27+ *Ans.*

(52) \$1.50×40=\$60 : \$60÷\$1.20=50 gal. : 50 gal.—40 gal. =10 gal. *Ans.*

(53) Int. of \$1 for 63 da.=\$.0105 : \$1.26÷\$.0105=\$120 *Ans.*

(54) In running 28 rd. the dog gains 3 rd., but the hare has 45 rd.=15 times 3 rd., the start; therefore the dog must run 28 times 15 rd.=420 rd., to catch the hare.

(55) $\frac{1}{5}$=20 % : \$1+\$.20=\$1.20 : \$1.50÷\$1.20=\$1.25, the cost ; \$2—\$1.25=\$.75 : \$1.25×.01=\$.0125 : \$.75÷\$.0125=60 % *Ans.*

(56) \$260+\$13=\$273 received for those sold : \$1.30×.40 =\$.52 : \$1.30+\$.52=\$1.82 : \$273÷1.82=150 bu. sold: \$260÷\$1.30=200 bu. : 200 bu.—150 bu.=50 bu. *Ans.*

(57) A is ahead 8 mi.×5=40 mi. : B gains 10—8=2 mi. per hour ; 40 mi.÷2 mi.=20 hr. *Ans.*

(58) A in 1 da. mows $\frac{2}{3}$ A. ; B $\frac{5}{6}$ A. ; and both can mow $\frac{2}{3}+\frac{5}{6}=1\frac{1}{2}$ A. in 1 da. : 9 A÷$1\frac{1}{2}$ A.=6 da. *Ans.*

(59) Both do 4 times as much as B : hence B could do it in 20×4=80 da, and A in $\frac{1}{3}$ of 80 da.=$26\frac{2}{3}$ da. *Ans.*

(60) 1 of the first lot cost $2\frac{1}{2}$ cts., and 1 of the 2d $2\frac{2}{3}$ cts. ; hence, 1 of each, that is, 2, cost $2\frac{1}{2}+2\frac{2}{3}=5\frac{1}{6}$ cts. : If 5 sold for 14 cts., 2 sold for $\frac{2}{5}$ of 14=$5\frac{3}{5}$ cts., $5\frac{3}{5}\div5\frac{1}{6}=\frac{13}{30}$ ct.=gain on 2 : $\frac{13}{30}$ ct. : 52 cts. : : 2 : 240 *Ans.*

(61) $2\frac{1}{2}=\frac{5}{2}$; $3\frac{1}{5}=\frac{16}{5}$; A can do it in $\frac{5}{2}$ da. and in $\frac{1}{2}$ da. he can do $\frac{1}{5}$, and in 1 da. $\frac{2}{5}$: similarly, B can do $\frac{5}{16}$ in 1 da. : $\frac{5}{16}+\frac{2}{5}=\frac{57}{80}$=what both do in 1 da. : $1\div\frac{57}{80}=1\frac{23}{57}$ da. *Ans.*

(62) As 2 min. : 3 min. : : 11 rods : $16\frac{1}{2}$, the space traveled by A in 3 minutes. Hence B, in traveling 17 rods, gains $\frac{1}{2}$ rod on A; and the object being to find how many rounds he must make to gain half a round, we have this proportion: As $\frac{1}{2}$ rod : 17 rods : : $\frac{1}{2}$ round : 17 rounds, the space to be gone over by B: consequently A will make $16\frac{1}{2}$ rounds.

(63) A does $\frac{1}{3}$ in 1 da., B $\frac{1}{5}$ and C $\frac{1}{4}-\frac{1}{5}=\frac{1}{20}$: all can do $\frac{1}{3}+\frac{1}{5}+\frac{1}{20}=\frac{7}{12}$ in 1 da. : $1\div\frac{7}{12}=1\frac{5}{7}$ da. *Ans.*

(64) From the question, it is obvious that in 1 day A and B do $\frac{1}{15}$, A and C $\frac{1}{12}$, and B and C $\frac{1}{10}$ of the work. By taking the sums of these fractions, we find *twice* what they all do in 1 day; that is, $\frac{1}{15}+\frac{1}{12}+\frac{1}{10}=\frac{4}{60}+\frac{5}{60}+\frac{6}{60}=\frac{15}{60}=\frac{1}{4}$, is twice the part of the work they all do in 1 day; here $\frac{1}{2}$ of $\frac{1}{4}=\frac{1}{8}$, is the part of the work they all do in 1 day: therefore they all do it in $1\div\frac{1}{8}=8$ days.

(65)

30 lb.	at 10 cts.	=	\$3.00
25	12	=	3.00
4	15	=	.60
50	20	=	10.00
109			)16.60(15$\frac{25}{109}$ cts. *Ans.*

(66) If 10 men can dig 1 acre in 11 days, then
1 man can dig 1 acre in $11\times10=110$ days,
and 1 man can dig $\frac{1}{110}$ of an acre in 1 day;
then 5 men can dig $\frac{1}{110}\times5=\frac{1}{22}$ of an acre in 1 day,
and 5 men can dig $\frac{1}{22}\times6=\frac{3}{11}$ of an acre in 6 days.
$6-\frac{3}{11}=\frac{66}{11}-\frac{3}{11}=\frac{63}{11}$ acres to be dug by boys.
If 18 boys can dig 1 acre in 11 days, then
1 boy can dig 1 acre in $11\times18=198$ days,
and 1 boy can dig $\frac{1}{198}$ of an acre in 1 day;
then 1 boy can dig $\frac{1}{198}\times6=\frac{1}{33}$ of an acre in 6 days.
But since $\frac{63}{11}$ acres are to be dug in 6 days by boys, each of whom can dig $\frac{1}{33}$ of an acre, it will require as many boys as $\frac{1}{33}$ acre is contained times in $\frac{63}{11}$ acres : $\frac{63}{11}\div\frac{1}{33}=\frac{63}{11}\times\frac{33}{1}=63\times3=189$ *Ans.*

(67) 24 in. = 2 ft. : $15+15+12+12=54$ ft., the distance around; 54 ft.×10 ft.=540 sq. ft. : 540 sq. ft.÷2 ft.= 270 ft = 90 yd. *Ans.*

(68) A could do it 3 times in 1 da., B 5 times and C once: therefore, the three could together do it 9 times in 1 da., and do it once in $\frac{1}{9}$ da. *Ans.*

(69) $28^2=784$: $784+50=834$ men *Ans.*

(70) $40^2=1600$: $1600+1600=3200$ sq. ft. : $\sqrt{3200}=56.56$ + ft. *Ans.*

(71) Suppose the work to consist of 150 equal parts, then the person does $\frac{3}{5}$ of $150=90$ parts in 30 days, or 3 parts in 1 day; $150-90=60$ parts done by both in 6 days; hence they both do $60\div6=10$ parts in 1 day; then $10-3=7$, the number of parts done by the assistant in 1 day: therefore $150\div7=21\frac{3}{7}$ days.

(72) $1-\frac{11}{15}=\frac{4}{15}=$B's share; $\frac{11}{15}-\frac{4}{15}=\frac{7}{15}$ of the estate=\$7420: $\frac{1}{15}=\frac{1}{7}$ of \$7420=\$1060, and the estate=\$1060×15=\$15900 *Ans.*

(73) $24^2=576$: $18^2=324$: $576+324=900$=square of the diagonal on the floor, this is the base in a triangle where the hypotenuse is sought. $12^2=144$: $900+144=1044$: $\sqrt{1044}=32.3+$ ft. *Ans.*

(74) 56 lb. tea $=2\frac{1}{2}\times56=140$ lb. coffee $=3\frac{1}{2}\times140=490$ lb. sugar : 10 cts.×490=\$49 *Ans.*

(75) From the question it is evident, that all three do $\frac{1}{12}$ of the work in 1 day, and that B in 1 day does $\frac{1}{24}$, and C $\frac{1}{34}$; hence A does $\frac{1}{12}-(\frac{1}{24}+\frac{1}{34})=\frac{1}{12}-(\frac{17}{408}+\frac{12}{408})=\frac{1}{12}-\frac{29}{408}=\frac{34}{408}-\frac{29}{408}=\frac{5}{408}$ in 1 day; therefore he can do the work alone in $1\div\frac{5}{408}=\frac{408}{5}=81\frac{3}{5}$ days. *Ans.*

(76) A built $\frac{4}{3}$ as much as C : B $\frac{3}{4}$ of $\frac{4}{3}$, that is, as much as C : $\frac{4}{3}+1+1=\frac{10}{3}$: A built $\frac{4}{3}$ out of $\frac{10}{3}$, that is, $\frac{4}{10}=\frac{2}{5}$: B and C each $\frac{3}{3}$ out of $\frac{10}{3}$, that is $\frac{3}{10}$: $\frac{2}{5}$ of 200 ft.=80 ft. A $\frac{3}{10}$ of 200 ft.=60 ft. B and C also 60 ft. *Ans.*

(77) $42.43^2=1800.3049$ square of hypotenuse: since the two sides are equal, $\frac{1}{2}$ of 1800.3049, or 900.15245 is the square of 1 side, which is the area. 900.15245 P.=5.62+A.

(78) If he had worked the whole time he would have earned \$1.25×25=\$31.25. Now, each day he was idle he lost \$1.75, that is, 50 cents which he spent, and \$1.25 which he did not earn. But he lost in all the difference between \$31.25 and \$19, which is \$12.25, and he lost this at the rate of \$1.75 per day; therefore he was idle as many days as 1.75 is contained times in \$12.25, which is 7; therefore he was idle 7, and consequently worked 18 da. *Ans.*

(79) B, who works 30 days, will do $\frac{1}{2}$ the work: hence A must do $\frac{1}{2}$, which will take him 20 days. $20-3=17$ days more : $30-17=13$ *Ans.*

(80) $1-\frac{5}{14}=\frac{9}{14}$: $\frac{2}{3}$ of $\frac{9}{14}=\frac{3}{7}$: $1-(\frac{5}{14}+\frac{3}{7})=\frac{3}{14}$=C's; $\frac{5}{14}-\frac{3}{14}=\frac{1}{7}=\$1300$. Property$=\$1300\times7=\$9100$: $\frac{5}{14}$ of $\$9100=\$3250$=A's; $\frac{3}{7}$ of $\$9100=\$3900$=B's; $\frac{3}{14}$ of $\$9100=\$1950$=C's *Ans.*

(81) By the first it receives $\frac{1}{3}$ in the hour, by the second $\frac{1}{5}$; by the two $\frac{1}{3}+\frac{1}{5}=\frac{8}{15}$; by the third $1-\frac{8}{15}=\frac{7}{15}$ in 1 hour; $1\div\frac{7}{15}=2\frac{1}{7}$ hr. *Ans.*

(82) \$1.20×60=\$72.00 : \$72×.10=\$7.20 : \$72+\$7.20=\$79.20 the amount to be received : \$1.50×20=\$30.00 : \$79.20—\$30=\$49.20 : 60—(20+5)=35 gal. : \$49.20÷35=$\$1.40\frac{4}{7}$ *Ans.*

(83) 1 apple cost $\frac{4}{3}$ cts., and sold for $\frac{3}{2}$ cts. : gain on 1 apple $=\frac{3}{2}-\frac{4}{3}=\frac{1}{6}$ ct. : 24 cts.$\div\frac{1}{6}$ ct.=144 *Ans.*

(84) \$4500×.018=\$81 : \$500—(\$25+\$81+\$134)=\$500—\$240=\$260, net income; \$4500×.01=\$45 : \$260÷\$45 $=5\frac{7}{9}$ % *Ans.*

(85) 60×30=1800 sq. ft. : 12 ft. 4 in.×8 ft. 4 in.=104 sq. ft. 10′ : 1800 sq. ft.—104 sq. ft. 10′=1695 sq. ft. 2′ : 1695 sq. ft. 2′×4=6780 sq. ft. 8′ *Ans.*

(86) \$700+\$600+\$400=\$1700 : $\frac{400}{1700}=\frac{4}{17}$: $\$260=\frac{4}{17}$ gain : $\frac{1}{17}=\frac{1}{4}$ of \$260=\$65 and gain=\$65×17=\$1105 *Ans.*

(87) 120×.10=12 : 12+8=20 : 20÷.05=400 *Ans.*

(88) \$120÷4=\$30 B's yearly debt : A, spending \$350 less, will save $\$350-\$30=\$320=\frac{1}{3}$ his annual income; \$320×3=\$960 *Ans.*

(89) $5^3\times.5236=65.45$: $65.45\times2=130.9$: $5^3=125$: $125\times2=250$: $130.9+250=380.9$: $\sqrt[3]{380.9}=7.24+$in. *Ans.*

(90) $\$1.00-\$.16\frac{2}{3}=\$.83\frac{1}{3}$: $\$25\div\$.83\frac{1}{3}=\$30$=cost of 1st. $\$30-\$25=\$5$ lost on the first, and therefore gained on the second. $\$5\div\$.16=\$31.25$=cost of 2nd.

(91) $1+\frac{1}{7}=\frac{8}{7}$; $\frac{8}{7}=24000$; $\frac{1}{7}=3000$ slain : $\frac{1}{2}$ of $3000=1500$ *Ans.*

(92) The 3d$=\frac{1}{4}$ of 3 times the 2nd$=\frac{3}{4}$ the 2nd. $1+3+\frac{3}{4}=\frac{4}{4}+\frac{12}{4}+\frac{3}{4}=\frac{19}{4}$, of which $\frac{4}{4}$ is $\frac{4}{19}$; $\frac{12}{4}$, $\frac{12}{19}$; and $\frac{3}{4}$, $\frac{3}{19}$; $\frac{4}{19}$ of 95 yd.=20 yd. 1st. : $\frac{12}{19}$ of 95 yd.=60 yd. 2nd : and $\frac{3}{19}$ of 95 yd.=15 yd. the 3d. *Ans.*

(93) $\$4\times60=\$240$: Amount of $\$1$ for 6 mon.$=\$1.03$; $\$240\div\$1.03=\$233.009+$: $\$250-\$233.009+=\$16.99+$ *Ans.*

(94) As often as he buys one cow, he buys 6 sheep and $\frac{6}{3}=2$ horses; 1 cow cost $\$30$; 6 sheep $\$30$; 2 horses $\$240$. While buying 1 cow, he expends $\$30+\$30+\$240=\$300$; $\$1500\div\$300=5$ cows; $5\times6=30$ sheep; $30\div3=10$ horses. *Ans.*

(95) 4 polls cost $\$4$: $\$328-\$4=\$324$, the tax on his property; $\$324\div.016=\$20250$ *Ans.*

(96) C does $1-\frac{5}{9}=\frac{4}{9}$; B does $\frac{2}{3}-\frac{4}{9}=\frac{2}{9}$ *Ans.*

(97) B received $\frac{1}{2}$ of A's$+\$100$; both received $\frac{3}{2}$ of A's$+\$100$, $\frac{3}{2}$ of A's$+\$100=\$1000$; $\frac{3}{2}$ A$=\$900$: A's$=\$600$, B's$=\$1000-\$600=\$400$ *Ans.*

(98) $\frac{2}{3}$ of B's$=\frac{3}{4}$ of A's$+\$50$; $\frac{1}{3}$ of B's$=\frac{3}{8}$ of A's$+\$25$, and B's$=\frac{9}{8}$ of A's$+\$75$, both have $\frac{17}{8}$ of A's$+\$75=\$500$: $\frac{17}{8}$ of A's$=\$425$: $\frac{1}{8}=\$25$: A's$=\$200$: B's$=\$500-\$200=\$300$ *Ans.*

(99) $\frac{1}{3}$ of the time past noon is equal to $\frac{1}{9}$ of the time to midnight, and the whole time past noon$=\frac{3}{9}$ or $\frac{1}{3}$ the time to midnight; the time from noon to midnight, which is 12 hr.$=1+\frac{1}{3}$, $\frac{4}{3}$ the time to midnight : $\frac{1}{3}=\frac{1}{4}$ of $12=3$ hr. [*Ans.*

(100) In the 4th store $\$\frac{1}{2}$ was $\frac{1}{2}$ her money; hence, on leaving the 3d store, she had $\$1$; before she spent the $\$1$, she had $\$2$=half what she had on entering the 3d store; hence, on leaving 2nd store, she had $\$4$; before spending $\$\frac{1}{2}$ she had $\$4\frac{1}{2}=\frac{1}{2}$ of $\$9$, her money on leaving 1st store; $\$9+\$1=\$10=\frac{1}{2}$ of $\$20$, her money at first.

KEY

TO

RAY'S SECOND BOOK,

CONTAINING

SOLUTIONS TO THE MORE DIFFICULT QUESTIONS.

ON TEACHING INTELLECTUAL ARITHMETIC.

It is to be regretted that many teachers are not acquainted with the great value of mental arithmetic. It is important, whether it be regarded as a mere mental discipline, or as the foundation of an accurate knowledge of arithmetic, and the various branches of mathematical science.

Whenever a pupil is found incapable of analyzing and solving, mentally, all the questions in Ray's 2d Book, the best course he can pursue is, at once, to study mental arithmetic, although in order to do so, it should be necessary to suspend, for a time, the study even of Algebra or Geometry. Persons more than twenty years old have studied it, who afterward declared that the time thus employed was productive of more beneficial results than twice the same amount of time ever employed at any other branch of education. It cannot, therefore, be too strongly recommended to every teacher who desires to render his pupils thorough scholars, to make them well acquainted with mental arithmetic.

It should be taught before written arithmetic. It often happens, however, that this subject has been neglected until after the pupil has commenced the study of written arithmetic. In such cases, the study of both may be attended to at the same time, one part of the recitation being devoted to mental, and the other to written arithmetic.

Method of Teaching.—*Classes.*—Pupils study best in classes; it is almost as easy for a teacher to instruct 15 pupils in a class, as one alone.

An important point is to keep the attention of the whole class directed to each question; one means of doing this is, for the teacher occasionally to read an example to the class, and then, having allowed sufficient time for the answer to be prepared, to call upon some one for it. By this means, each one is obliged to solve the example mentally, as he may be called upon to answer it. It is sometimes proper to put the same example to the whole class, giving all permission to answer it; this plan succeeds better in small classes than in large, as it has a tendency to produce noise and confusion.

When the class meets to recite, two methods may be followed in recitation. *First*, each pupil may have his book open, and, when his turn comes, read his question, and then proceed to analyze and answer it. Or, *Second*, the class may close books, and the teacher read each question distinctly, and then call upon a pupil to analyze and answer it, taking the pupils in succession.

Both methods are practiced by good teachers; the first, however, is especially applicable to small scholars, while the latter answers very well with those who are older; it is an excellent means of teaching pupils to command their attention—a very important acquisition.

ARITHMOMETER.—Some difference of opinion exists among teachers with regard to the extent to which artificial aid should be given to pupils in the study of mental arithmetic. It is certain that visible illustrations may be used, in some cases, to great advantage, especially to give clear and precise ideas in the commencement of a subject; but, it is also true, on the other hand, that a frequent use of artificial aid tends to prevent the pupil from exercising his own intellectual powers, and thus, if carried too far, is productive of positive injury. It should be recollected, that the younger the pupils, the more they require such assistance as can be given them from tangible objects.

The superior advantage of the arithmometer consists in this, that it can be seen by the whole class at the same time, and it is so simple in its construction that it can be made by any mechanic. The arithmometer consists of a square frame with 12 strong pieces of wire passing from one side to the other. On each piece of wire there are 12 balls, one-half black, the other half white, so as to form a contrast, and alternately disposed, so as to be easily counted. As the arithmometer contains 144 balls, it may be used to represent all numbers, from one up to this amount, so as to give pupils an idea of the relative magnitude of numbers.

The operations of addition and subtraction are so easily shown by it, that any explanation is unnecessary. To represent multiplication, for instance, the multiplication of 4 by 3; take 3 rods, each having 4 balls on the one end of it, (the other balls being pushed out of the way toward the other end,) and the pupils will easily see that 3 times 4 make 12; in the same manner it may be shown that 4 times 3 are 12; and hence, that the product of two numbers is the same which ever be made the multiplier.

Division is shown by taking the number of balls represented by the dividend, and separating them into parts, each part containing as many balls as the divisor contains units; the number of parts, into which the dividend is thus divided, will represent the quotient: thus, to divide 15 by 3, take 15 balls and separate them into parts, each containing 3 balls, and it will be seen that there are 5 divisions, so that 15 contains 3 five times.

The operations in fractions, so far as they can be explained without dividing a unit, are also easily shown by it. To give pupils the first ideas of fractions, the division of an apple is preferable to any thing else, it being an operation with which pupils are generally familiar.

Analysis.—Among the modern improvements in the art of teaching, the analytic and inductive method, as applied to arithmetic, forms one of the most important. To analyze, means to separate into elementary or first principles. Intellectual or mental is used to designate that kind of arithmetic, the operations of which are performed wholly in the mind. This kind of arithmetic is quite as valuable for the habits of close thinking and accurate reasoning, as for the mere amount of knowledge which is acquired.

To enable the pupil to realize to the fullest extent the advantages of the study, he should clearly understand every process, and be able to give a reason for every operation. It can not, therefore, be too strongly recommended to the teacher, to require every question to be solved by methods purely analytical, which the pupil can both comprehend and explain. In this respect, the pupil will derive great advantage from a careful attention to the solutions termed ANALYSIS, (Ray's Arithmetic, 2d Book,) which are intended as *Model Solutions.*

To explain the difference between an Analytical Solution and one that is not, take the following question.

What is three-fourths of 12?

Solution. 3 times 12 are 36, and 4 in 36 goes 9 times. *Ans.* 9.

Analytical Solution. 3-fourths are 3 times 1-fourth; 1-fourth of 12 is 3, and 3 times 3 are 9: therefore 9 is 3-fourths of 12.

In analyzing we must generally refer to unity, and the pupil should always bear this in mind: thus, if 2 yards of cloth cost 6 dollars, what will 5 yards cost? Here we have given the value of 2 yards, and must find the value of *one* yard, after which we can readily find the value of 5 yards.

Again, suppose we have this question: 2-thirds of a certain number is 8, what is 5-thirds of the same number?

ANALYSIS.—1-*third is* 1 *half of* 2-*thirds; if* 2-*thirds are* 8, 1-*third is* 1-*half of* 8, *which is* 4; *if* 1-*third is* 4, *then* 5-*thirds are* 5 *times* 4, *which are* 20. *Ans.* 20.

If learners pursue the method taught in the book, they will have no difficulty in progressing; but when they depart from it, they will soon be involved in perplexity. The questions are so arranged, that each one depends, in some measure, on that which precedes it; consequently, pupils should understand every thing as they proceed.

When a teacher finds his class unable to advance, he will always find, that they are not sufficiently acquainted with the preceding part. In such cases review. The pupils should go through the first six sections at least two or three times before entering upon the seventh and succeeding sections. If they then properly understand multiplication and division, they will find little difficulty in the remainder of the work.

ORAL EXERCISES IN MENTAL ARITHMETIC.

The following method of exercising pupils mentally, in Addition, Subtraction, Multiplication, and Division, will be found to have the happiest effect in concentrating the attention of the class, and exciting a lively interest in the exercises. It also renders pupils remarkably expert in the performance of the operations.

The success of the whole, however, depends upon the instructor; he must propose the exercises *orally*, and carry on the operations in his own mind, so as not to give examples that are either too difficult or improper, and also to be able to decide on the correctness of the result.

The instructor should announce the exercises clearly, distinctly, and deliberately; while this is being done, each pupil should carry on the operations in his own mind, so as to be able to give the result almost as soon as the announcement is closed. To avoid confusion in the class, when a pupil thinks he has obtained the correct result, let him hold up his hand. If his answer is incorrect, let some other pupil who has obtained a different result, announce it; and so on, until a correct answer is obtained.

Following will be found specimens of various exercises. These are given to show teachers *what kind of exercises* may be employed, but not for the purpose of being presented to the class, for let it be distinctly understood, that the success of the whole depends largely on the teachers announcing the exercises *orally.*

In the following specimens the exact language is given which the instructor should use. It should always be brief as possible, no more words being used than are necessary to point out the exercises to be performed.

1. Five, add 6, add 3, add 4, add 2, add 9, add 7. Result? (36).

The meaning of this is, that the pupil is to take 5, then add to it 6, then to this sum add 3, and so on. Lastly, for the sake of brevity, the teacher says *Result?* instead of *What is the result?*

2. Four, add 5, subtract 2, add 7, add 5, subtract 6, add 2. Result? (15).

3. Seven, add 9, add 8, add 7, add 3, add 2, add 6, add 3, add 9, add 6. Result? (60).

4. Eight, add 5, subtract 3, subtract 4, subtract 7,—class says can not be done, before the teacher has time to ask for the result, or to propose any thing additional.

5. Twelve, subtract 3, subtract 5, add 6, subtract 7, add 9. Result?

6. Fourteen, subtract 8, add 6, add 7, add 5, subtract 9, add 4, add 7. Result?

Exercises involving only addition and subtraction, are less interesting than those which also combine multiplication and division. Such exercises should therefore be used as soon as the pupil is acquainted with these processes. No division should be required that leaves a remainder.

7. Three, multiply by 2, add 4, multiply by 3, divide by 5. Result? (6).

8. Four, multiply by 3, add 3, divide by 5, add 6, multiply by 2, add 4. Result? (22).

9. Seven, multiply by 3, subtract 5, divide by 4, add 1, multiply by 8, divide by 10. Result? (4).

10. Nine, multiply by 6, subtract 4, divide by 10, add 3, multiply by 4, subtract 5, divide by 9. Result? (3).

11. Six, multiply by 5, add 5, divide by 7, subtract 5, add 4, multiply by 10, add 8, divide by 6, subtract 6. Result? (2).

12. Eleven, add 5, divide by 8, add 5, multiply by 4, add 5, divide by 11, add 5, divide by 4, add 5. Result? (7).

13. Twenty-one, divide by 3, add 3, multiply by 3, subtract 3, divide by 3. Result? (9).

14. Sixty, subtract 4, divide by 7, add 3, multiply by 4, subtract 2, divide by 6, add 5, multiply by 6, divide by 9, add 2, divide by 2, subtract 2. Result? (3).

15. Twelve, multiply by 5, add 3, divide by 9, multiply by 7, add 5, divide by 6, add 1, multiply by 10, add 8, divide by 9, subtract 5, multiply by 6, add 3, divide by 5. Result? (9).

16. Eight, multiply by 8, add 8, divide by 8, subtract 8. Result? (1).

17. Seven, multiply by 7, add 7, divide by 7, subtract 7. Result? (1).

The length and difficulty of the exercises must be graduated by the age, attainments, and ability of the members of the class; if too easy they will not excite sufficient interest, and if too difficult, the class will be discouraged. At first, the examples should be short, afterward they may be longer, like the following:

18. Fifteen, add 5, multiply by 3, add 4, divide by 8, subtract 2, multiply by 7, add 12, divide by 9, add 3, multiply by 10, add 6, divide by 12, multiply by 8, subtract 1, divide by 7, subtract 7. Result? (2).

By teaching the pupils the meaning of the terms *square, square root, cube, and cube root,* the exercises may be made to embrace the operations of squaring, extracting the square root, etc., as in the following exercises. For the sake of brevity, the terms square root, and cube root, are used to denote that the square root, or cube root, is to be extracted.

19. Three, square, add 7, square root, add 3, square, subtract 1, divide by 6, add 1, square root, multiply by 8, add 1, square root. Result? (5).

20. Seven, multiply by 4, subtract 1, cube root, add 5, cube root, multiply by 12, divide by 3, subtract 4, cube, subtract 1, divide by 9, multiply by 4, subtract 1, cube root. Result? (3).

21. Four, square root, cube, add 1, square root, cube, add 9, square root, subtract 2, cube, square root, divide by 2, add 1, cube, subtract 4, square root, add 5, square root, add 5, multiply by 7, add 1, cube root, square root. Result? (2).

To TEACHERS.—In the following solutions, for the sake of brevity, arithmetical signs are sometimes used; instructors, however, in explaining solutions to pupils, should always employ language the most simple and easily comprehended.

A full analysis is given to some of the more difficult examples, while to others less difficult, the solution is as brief as possible.

In all cases the pupil should be required to give a full analysis; the style and manner of doing this being a better test of scholarship than the ability to give merely a correct answer.

SECTION 10.—LESSON IV.

(6) If each man received 1-sixth, there must have been 6 men; hence each man received 1-sixth of 42 dollars, which is 7 dollars.

(10) One-fifth of 20 cents is 4 cents, what she received from her brother; 1-half of 4 cents is 2 cents, what she received from her sister; hence, she received, in all, 20 cents, and 4 cents, and 2 cents=26 cents.

(12) If he gave 4-sevenths, he had 3-sevenths left; 1-seventh of 14 cents is 2 cents, and 3-sevenths are 3 times 2 cents, which are 6 cents.

(14) One-fifth of 30 yards is 6 yards, and 2-fifths are 2 times 6 yards, which are 12 yards; if 12 yards cost 48 dollars, 1 yard will cost 1-twelfth of 48 dollars, which is 4 dollars.

(15) One-fifth of 25 cents is 5 cents, and 3-fifths are 3 times 5 cents, which are 15 cents: at 2 cents each, 15 cents will buy as many peaches as 2 cents is contained times in 15 cents, which is 7 and 1-half. *Ans.* $7\frac{1}{2}$ peaches.

(16) One-ninth of 54 is 6, and 5-ninths are 5 times 6, which are 30. To give each girl 1 chestnut would require 3 chestnuts; hence each girl received as many chestnuts as 3 chestnuts is contained times in 30 chestnuts, which is 10. *Ans.* 10 chestnuts.

(18) One yard cost 1-seventh of 7 yards; if 7 yards cost 42 dollars, 1 yard cost 1-seventh of 42 dollars, which is 6 dollars, and 3 yards would cost 3 times 6 dollars, which are 18 dollars. If 9 barrels of cider cost 18 dollars, 1 barrel would cost 1-ninth of 18 dollars, which is 2 dollars.

(22) One-fourth of 28 is 7; 1-seventh of 28 is 4, and 3-sevenths are 3 times 4, which are 12; 7 cents and 12 cents are 19 cents, which taken from 28 cents, leaves 9 cents. If 3 books cost 9 cents, 1 book will cost 1-third of 9 cents, which is 3 cents.

SECTION 11.—LESSON III.

(13) Since 5-sevenths are apple trees, the remaining 2-sevenths are cherry trees. One-seventh is 1-half of 2-sevenths; if 2-sevenths are 12, one-seventh is 1-half of 12, which is 6, and if 1-seventh is 6, 5-sevenths are 5 times 6, which are 30, the number of apple trees.

(17) Since 25 is 5-eighths, 1-fifth of 25, which is 5, is 1-eighth; hence, 8-eighths is 8 times 5 years, or 40 years, the father's age. One-tenth of 40 years is 4 years, the age of the youngest.

(18) Since 45 is 5-thirds, 1-fifth of 45, which is 9, is 1-third of the first cost, and 3 times 9, which is 27, is the first cost; hence, he gained 45 less 27, which is 18 dollars. Or, the gain is evidently 2-thirds of the first cost, which is twice 9 dollars, or 18 dollars.

(21) Since he gained 2-sevenths, he must have sold it for 9-sevenths of the first cost; therefore, 45 is 9-sevenths, and 1-ninth of 45, which is 5, is 1-seventh of the first cost; hence, 7 times 5, which are 35, is the first cost.

SECTION 12.—LESSON I.

(7) One-fifth of 20 is 4, and 2-fifths are 2 times 4, which are 8; 1-third of 15 is 5, and 8 is once 5, and 3-fifths of 5.

The following are the answers to the remaining examples in this lesson. Although they are explained in the form of common fractions, the pupil should give them after the manner expressed in the Arithmetic; thus, the answer to question 8 should be read, *two times 5 and 2-fifths of 5.*

(8) $2\frac{2}{5}$. **(9)** $3\frac{1}{8}$. **(10)** $11\frac{2}{3}$. **(11)** $4\frac{3}{6}$. **(12)** $6\frac{1}{4}$.
(13) $1\frac{2}{6} : 2\frac{5}{8}$. **(14)** $4 : 3\frac{3}{9}$ **(15)** $4\frac{2}{10}$. **(16)** $7\frac{2}{10}$.

LESSON II. *Answers.*

(2) 9. **(3)** 18. **(4)** 15. **(5)** 9. **(6)** 40.
(7) 35. **(8)** 44. **(9)** 4. **(10)** 2. **(11)** 1.
(12) 6. **(14)** $2\frac{4}{5}$. **(15)** $7\frac{3}{6}$. **(16)** $3\frac{3}{5}$. **(17)** $7\frac{5}{7}$.
(18) 5. **(19)** $5\frac{5}{6}$. **(20)** $7\frac{3}{6}$. **(21)** $3\frac{3}{10}$. **(22)** $1\frac{4}{8}$.

(23) 1-tenth of 50 is 5, and 7-tenths are 7 times 5, which are 35; if 35 is 5-ninths, 1-fifth of 35, which is 7, is 1-ninth, and 9-ninths, or the number, is 9 times 7, which are 63. If 8 is 4-fifths, 1-fourth of 8, which is 2, is 1-fifth, and 5-fifths, or the number, are 5 times 2, which are 10. 63 is as many times 10 as 10 is contained times in 63, which are 6 times 10 and 3-tenths of 10. *Ans.* 6 times and 3-tenth times.

(24) *Ans.* 4 times and 2-sevenths of a time.

(25) 24. **(26)** 12. **(27)** 2-thirds of 21. **(28)** 18.

(29) 40. **(30)** 3 times 7.

SECTION 13.—LESSON I.

(25) In 42 there are 6 *sevens*, and Joseph husks as many *threes* as his father husks *sevens*, that is, 6 *threes*, which are 18 rows.

(28) If the horses eat 16 bushels in 2 weeks, they will eat 1-half of 16 bushels, which is 8 bushels, in 1 week; if they eat 8 bushels in 1 week, it will require them as many weeks to eat 56 bushels as 8 bushels are contained times in 56 bushels, that is 7. *Ans.* 7 weeks.

It is easily seen, that the *number* of horses has nothing to do with the solution. The question would be the same if the words, *a certain number*, were substituted for 5.

(30) To eat it in 1 day would require 6 times 12=72 horses; and to eat it in 9 days, $\frac{1}{9}$ of 72=8 horses *Ans.*

(31) If 1 lb. of sugar=2 lb. of rice, 1 lb. of sugar and 2 lb. of rice=4 lb. of rice=1 lb. of coffee *Ans.*

LESSON II.

(22) *Ans.* 4. **(23)** To Thomas 1 the most, and he had 6 left.

(24) *Ans.* 5 dollars. **(25)** *Ans.* 54 dollars.

(26) By subtracting 1-third and 2-fifths of 15 from it, there are left 4 yards. The cost was 30 dollars, and the sales 58 dollars. Difference, 28 dollars.

(27) *Ans.* 1 dollar.

(28) 23 less 18 is 5, the number of miles per day that B gains on A. If it requires 1 day to gain 5 miles, it will require 8 days to gain 40 miles. *Ans.* 8 days.

(29) 10 less 7=3 yd. gain in 1 min. The hound will gain 90 yd. in 90÷3=30 min., during which time the hound goes 30 times 10=300 yd., and the hare 30 times 7 =210 yards.

(30) 7 less 4=3 rods. The hound runs 7 rods to gain 3 rods, and will run 35 rods, which is 5 times 7, to gain 5 times 3 rods=15 rods *Ans.*

(31) 3 miles less 1 mile is 2 miles; the first gains 2 miles in going 3 miles, and will gain 4 miles, or twice two miles, in going twice 3=6 miles *Ans.*

(32) Since D gains 3 miles on C in traveling 5, and since 15 is 5-*threes*, it will require him to travel 5-*fives*, that is, 25 miles, to overtake C.

(33) Since 8 gallons are *filled*, and 5 gallons *emptied* in 1 hour, it is evident the cistern is filled at the rate of 3 gallons (8 less 5 equals 3) per hour; hence it will require as many hours to fill it as 3 gallons is contained times in 24 gallons, that is 8. *Ans.* 8 hours.

(34) Since it is filled by the first pipe in 6 hours, it is filled at the rate of 6 (36÷6=6) gallons per hour; and since it is emptied in 9 hours, it is emptied at the rate of 4 (36÷9=4) gallons per hour; hence when both pipes are left open, it is filled at the rate of 2 (6—4=2) gallons per hour; consequently it will be filled in 36÷2=18 hours.

(35) 8 is 2-fifths of 20 dollars, the price of the coat, and 1-half of 8 is 4 dollars, the price of the vest; hence, the whole cost 8 and 20 and 4 dollars, that is, 32 dollars.

SECTION 19.—LESSON III.

(11) It requires $8\times 11=88$ hours to perform the work; 88 hours÷10 hours=$8\frac{8}{10}$ days of 10 hours each.

(12) 8 miles for 37 cents would be $\frac{37}{8}$ cents for 1 mile, and 11 times $\frac{37}{8}$ is $\frac{407}{8}=50\frac{7}{8}$ cents for 11 miles.

(13) If it requires 2 pipes to empty it in 17 minutes, it will take 1 pipe twice as long, that is, 34 minutes, and 3 pipes 1-third as long as 1 pipe, that is, 1-third of 34 minutes=$11\frac{1}{3}$ minutes.

(14) 1 horse will require 1-fifth as much as 5 horses, and 7 horses 7 times as much as 1 horse. 1-fifth of 18 bushels is $\frac{18}{5}$, and 7 times $\frac{18}{5}$ are $\frac{126}{5}$, which are $25\frac{1}{5}$ bushels.

(18) $\frac{2}{7}$ of $18=\frac{36}{7}=5\frac{1}{7}$ feet in the earth; and $\frac{5}{7}$ of $18=\frac{90}{7}$ $12\frac{6}{7}$ feet in the air.

(19) $\frac{2}{9}$ of $15=\frac{30}{9}=3\frac{1}{3}$, A's part. $\frac{1}{3}$ of $15=5$, B's part. $5+3\frac{1}{3}=8\frac{1}{3}$, and $15-8\frac{1}{3}=6\frac{2}{3}$, C's part.

(20) 48 dollars. **(21)** 8 cents.

(22) 12. **(23)** $1\frac{1}{2}$ dollars.

SECTION 20.—LESSON II.

(13) It will require 1 man twice as long as 2 men, and 3 men 1-third as long as 1 man. Twice $3\frac{1}{3}$ days are $6\frac{2}{3}$ days, the time in which 1 man can do it; and $\frac{1}{3}$ of $6\frac{2}{3}$ days is $\frac{1}{3}$ of $\frac{20}{3}$ days, which are $\frac{20}{9}$ days, equal to $2\frac{2}{9}$ days, the time in which 3 men can do it.

(14) *Ans.* $5\frac{5}{21}$ days. **(15)** *Ans.* $9\frac{3}{4}$ days.

SECTION 22.—LESSON I.

(34) Since he gained 1-fourth, he must have sold it for 5-fourths of the first cost. If 18 is 5-fourths, 1-fifth of 18, which is $\frac{18}{5}$, is 1-fourth, and 4-fourths, or the whole, are 4 times $\frac{18}{5}$, which are $\frac{72}{5}$, equal to $14\frac{2}{5}$ dollars.

(35) *Ans.* $20\frac{5}{6}$ dollars cost; gain $4\frac{1}{6}$ dollars.

(36) *Ans.* $4\frac{1}{2}$ dollars. **(37)** *Ans.* $3\frac{3}{4}$ dollars.

LESSON II.

(6) If 4-fifths of a yard cost 8 cents, 1-fifth will cost 1-fourth of 8 cents, which is 2 cents, and 5-fifths, or a yard, will cost 5 times 2 cents, which are 10 cents. At 10 cents a yard, 25 cents will buy as many yards as 10 cents is contained times in 25 cents, which is $2\frac{1}{2}$ times.

Ans. $2\frac{1}{2}$ yards

(7) *Ans.* $6\frac{2}{5}$ dollars.

(8) If 6-sevenths of a yard cost 4 dollars, 1-seventh will cost 1-sixth of 4 dollars, which is $\frac{4}{6}$ or $\frac{2}{3}$ of a dollar, and 7-sevenths, or a yard, will cost 7 times $\frac{2}{3}$, which are $\frac{14}{3}$ dollars. 12 dollars will buy as many yards as $\frac{14}{3}$ dollars are contained times in 12 dollars. But $\frac{14}{3}$ is contained in 12 as often as $\frac{14}{3}$ is contained in $\frac{36}{3}$; and $\frac{14}{3}$ is contained in $\frac{36}{3}$ as often as 14 is contained in 36; 14 in 36 is contained $2\frac{8}{14}=2\frac{4}{7}$ times. *Ans.* $2\frac{4}{7}$ yards.

(10) *Ans.* $3\frac{1}{5}$ yards.

(11) If he gained 1-third, he must have sold it for 4-thirds of what it cost. If 12 is 4-thirds, 1-fourth of 12, which is 3, is 1-third, and 3-thirds, or the first cost, is 3 times 3, which are 9 dollars. If 5 yards cost 9 dollars, 1 yard cost $\frac{1}{5}$ of 9 dollars, which is $1\frac{4}{5}$ dollars.

(12) If he gained 2-fifths, he must have sold it for 7-fifths of the cost. If 21 is 7-fifths, 1-seventh of 21, which is 3, is 1-fifth, and 5-fifths, or the first cost, is 5 times 3, which are $15. This at 3-fifths of a dollar a bushel, would buy as many bushels as 3-fifths of a dollar is contained times in $15, which is 25 times. *Ans.* 25 bu.

(13) If $\frac{3}{5}$ cost $\frac{2}{3}$ of a cent, $\frac{1}{5}$ will cost $\frac{2}{9}$ of a cent, and one whole apple $\frac{10}{9}$ of a cent; and 3 apples cost $\frac{30}{9}=3\frac{1}{3}$ cents.

(14) $2\frac{3}{4}=\frac{11}{4}$: $5\frac{1}{2}=\frac{22}{4}$: If $\frac{11}{4}$ of a yard cost $4\frac{3}{5}$ dollars, $\frac{22}{4}$, which is twice $\frac{11}{4}$, will cost twice $4\frac{3}{5}=9\frac{1}{5}$ dollars.

(15) Then 8 dollars must be $\frac{4}{5}$ of the cost; $\frac{1}{5}$ of the cost is 2 dollars, and the cost is 5 times 2 dollars, which are 10 dollars; 11 dollars less 10 dollars will be 1 dollar, which is $\frac{1}{10}$ the cost.

SECTION 25.—LESSON I.

(3) $\frac{1}{2}$ of $3=\frac{3}{2}$, $5\times\frac{3}{2}=\frac{15}{2}$; $\frac{1}{3}$ of $\frac{15}{2}=\frac{5}{2}$, and $2\times\frac{5}{2}=\$5$ *Ans.*

(4) $\frac{1}{4}$ of $24=6$, $7\times6=42$; $\frac{1}{14}$ of $42=3$, $5\times3=15$ cts. *Ans.*

(5) $\frac{1}{5}$ of $15=3$, $9\times3=27$; $\frac{1}{2}$ of $27=\$13\frac{1}{2}$ *Ans.*

(6) $\frac{1}{3}$ of $30=10$, $8\times10=80$; $\frac{1}{16}$ of $80=5$, $7\times5=35$ trees. *Ans.*

(7) $1\frac{2}{5}=\frac{7}{5}$, $\frac{1}{7}$ of $14=2$, $5\times2=10$; $2\frac{1}{2}\times10=\$25$ *Ans.*

(8) $1\frac{1}{2}=\frac{3}{2}$, $5\frac{1}{4}=\frac{21}{4}$; $\frac{1}{3}$ of $\frac{21}{4}=\frac{7}{4}$, 2 times $\frac{7}{4}=\frac{7}{2}$, the cost of 1 barrel, and $2\frac{1}{2}$ times $\frac{7}{2}=\frac{35}{4}=8\frac{3}{4}$ dollars *Ans.*

(9) $3\frac{1}{3}=\frac{10}{3}$, $\frac{1}{10}$ of $20=2$, and 3 times $2=6$; $2\frac{5}{6}$ times $6=17$ cts. *Ans.*

(10) $3\frac{3}{4}=\frac{15}{4}$, $\frac{1}{15}$ of $30=2$, 4 times $2=8$ miles, the distance traveled in 1 hour; $7\frac{1}{4}$ times $8=58$ miles. *Ans.*

(11) $\frac{1}{10}$ of $1\frac{1}{4}=\frac{1}{8}$, 11 times $\frac{1}{8}=1\frac{3}{8}$ dollars. *Ans.*

(12) $\frac{1}{6}$ of $9\frac{3}{5}=\frac{8}{5}$, and $\frac{1}{8}$ of $\frac{8}{5}=\frac{1}{5}$; 9 times $\frac{1}{5}=\frac{9}{5}$, and 7 times $\frac{9}{5}=12\frac{3}{5}$ dollars. *Ans.*

(13) $5\frac{3}{4}=\frac{23}{4}$, $9\frac{1}{5}=\frac{46}{5}$, and $3\frac{2}{3}=\frac{11}{3}$. $\frac{1}{23}$ of $\frac{46}{5}=\frac{2}{5}$, and 4 times $\frac{2}{5}=\frac{8}{5}$; $\frac{1}{3}$ of $\frac{8}{5}=\frac{8}{15}$, and 11 times $\frac{8}{15}=\frac{88}{15}=5\frac{13}{15}$ dollars. *Ans.*

(14) $\frac{1}{5}$ of $8\frac{1}{3}=\frac{5}{3}$, which is $\frac{1}{7}$ of the number, and 7 times $\frac{5}{3}=\frac{35}{3}$, and $\frac{4}{5}$ of $\frac{35}{3}=\frac{28}{3}=9\frac{1}{3}$. *Ans.*

(15) $2\frac{2}{3}=\frac{8}{3}$: $3\frac{1}{2}=\frac{7}{2}$: $2\frac{1}{2}=\frac{5}{2}$; $\frac{1}{8}$ of $\frac{7}{2}=\frac{7}{16}$, and 3 times $\frac{7}{16}=\frac{21}{16}$, or the number. $2\frac{1}{2}$ times $\frac{21}{16}=\frac{21}{16}\times\frac{5}{2}=\frac{105}{32}=3\frac{9}{32}$. *Ans.*

(16) $\frac{1}{3}$ of $4\frac{1}{2}=1\frac{1}{2}$, and 4 times $1\frac{1}{2}=6$ dollars, the price of a barrel; $\frac{1}{5}$ of $6=\frac{6}{5}$, and 3 times $\frac{6}{5}=\frac{18}{5}=3\frac{3}{5}$ dollars. *Ans.*

(17) $\frac{1}{2}$ of $\frac{3}{5}=\frac{3}{10}$, and 3 times $\frac{3}{10}=\frac{9}{10}$; $\frac{1}{6}$ of $\frac{9}{10}=\frac{3}{20}$, and 5 times $\frac{3}{20}=\frac{3}{4}$ dollars. *Ans*

(18) $\frac{1}{5}$ of $30=6$, and $\frac{1}{3}$ of $6=2$, the wages of *one* man *one* day 4 times $2=8$, and 7 times $8=56$ dollars. *Ans*

(19) $\frac{1}{6}$ of $36=6$, and $\frac{1}{8}$ of $6=\frac{6}{8}=\frac{3}{4}$. 5 times $\frac{3}{4}=\frac{15}{4}$, and 12 times $\frac{15}{4}=45$ dollars. *Ans*

(20) $\frac{1}{3}$ of $12=4$, $\frac{1}{8}$ of $4=\frac{1}{2}$; 5 times $\frac{1}{2}=\frac{5}{2}$, and 3 times $\frac{5}{2}=7\frac{1}{2}$ rods. *Ans*

(21) $\frac{1}{6}$ of $36=6$, $\frac{1}{10}$ of $6=\frac{3}{5}$; 5 times $\frac{3}{5}=3$, and 9 times $3=27$ bushels. *Ans.*

(22) If 5 oxen eat 2 acres in 6 days, they will eat 1 acre in 3 days; and 1 ox will eat 1 acre in 5 times 3 days, or 15 days. Then it will require 1 ox 8 times 15 days, which are 120 days, to eat 8 acres; and it will require 12 oxen $\frac{1}{12}$ of 120 days, which are 10 days, to eat 8 acres.

Ans. 10 days.

(23) 400 dollars for 5 months is 80 dollars for 1 month, and if 8 persons spend 80 dollars in 1 month, *one* person will spend 10 dollars in 1 month. Hence 11 persons will spend $110 in 1 month, and in 8 months $880. *Ans.*

(24) 10 oxen on 5 acres for 3 months, is
2 oxen on 1 acre for 3 months; or
6 oxen on 1 acre for 1 month; or
90 oxen on 15 acres for 1 month; or
18 oxen on 15 acres for 5 months; or
126 sheep on 15 acres for 5 months. *Ans.* 126.

LESSON II.

(1) 5 times 18=90; $\frac{1}{9}$ of 90=10. *Ans.* 10 men.

(2) 8 times 15=120; $\frac{1}{12}$ of 120=10. *Ans.* 10 men.

(3) 9 times $2\frac{1}{2}=22\frac{1}{2}$; $\frac{1}{5}$ of $22\frac{1}{2}=4\frac{1}{2}$. *Ans.* $4\frac{1}{2}$ hr.

(4) To do the work in $\frac{1}{5}$ of the time will require 5 times as many men; 5 times 5 men are 25 men; twice as large will require twice as many men; twice 25 men are 50 men. *Ans.* 50 men.

(5) 8 times 5=40; $\frac{1}{5}$ of 40=8. *Ans.* 8 days.

(6) If 6 men can do the whole work in 5 days, they will do one-half of it in $2\frac{1}{2}$ days. The whole work requires 6 times 5=30 days work of one man, and half of it will require 15 days work of one man. Since 9 men (6+3) are employed to finish it, it will require $\frac{1}{9}$ of 15 days, or $1\frac{2}{3}$ days. Hence the whole time is $2\frac{1}{2}+1\frac{2}{3}=4\frac{1}{6}$ days.

(7) It will require 28 days work of one man, and 7 men will do half the work, or 14 days work of one man in 2 days; and since 3 men then leave, there are 7 less 3, or 4 men left, to do 14 days work of one man, which they will do in $\frac{1}{4}$ of 14 days, or $3\frac{1}{2}$ days. Hence the whole time is $2+3\frac{1}{2}=5\frac{1}{2}$ days.

(8) 5 times 8=40, the number of cent loaves. $\frac{1}{4}$ of 40=10, the number of 4 cent loaves. $\frac{1}{10}$ of 40=4, the number of 10 cent loaves.

(10) 3 times 20=60, the number of cent loaves, and $\frac{1}{4}$ of 60 =15, the number of 4 cent loaves. $\frac{1}{5}$ of 60=12, the number of 5 cent loaves.

(11) 3 times $8=24$, the weight in oz., when flour is \$1 a barrel. $\frac{1}{2}$ of $24=12$; $\frac{1}{4}$ of $24=6$, the weight respectively, when flour is 2 and 4 dollars a barrel.

(13) $\$3\frac{1}{2}=\$\frac{7}{2}$: $\$2\frac{1}{2}=\$\frac{5}{2}$: 7 times $7=49$ oz , when flour is $\$\frac{1}{2}$ per barrel : $\frac{1}{5}$ of $49=9\frac{4}{5}$ oz., the weight when flour is $\$\frac{5}{2}$ per barrel. *Ans.* $9\frac{4}{5}$ oz.

(14) 1 man could mow it in 6 times $5\frac{1}{2}=33$ days, and 10 men in $\frac{1}{10}$ of $33=3\frac{3}{10}$ days : $5\frac{1}{2}-3\frac{3}{10}=2\frac{1}{5}$ days. *Ans.*

LESSON III.

(2) $3+4+5=12$. $\frac{3}{12}$ of $36=9$, W's; $\frac{4}{12}$ of $36=12$, T's; $\frac{5}{12}$ of $36=15$, John's share.

(3) $\frac{2}{3}$ of $7\frac{1}{2}=5$, share of 1st; $\frac{1}{3}$ of $7\frac{1}{2}=2\frac{1}{2}$, share of 2nd.

(4) $40-25=15$, what B paid. $\frac{25}{40}=\frac{5}{8}$, and $\frac{5}{8}$ of $56=35$ dollars, A's share. $\frac{15}{40}=\frac{3}{8}$, and $\frac{3}{8}$ of $56=21$ dollars, B's share.

(5) $\frac{1}{8}+\frac{1}{4}=\frac{3}{8}$, and $1-\frac{3}{8}=\frac{5}{8}$, C's part. $\$860-\$500=\$360$: $\frac{1}{8}$ of $360=45$ dollars, A's loss; $\frac{1}{4}$ of $360=90$ dollars, B's loss; $\frac{5}{8}$ of $360=225$ dollars, C's loss.

(6) $\frac{2\frac{1}{2}}{7}=\frac{5}{14}$, and $\frac{4\frac{1}{2}}{7}=\frac{9}{14}$; $\frac{5}{14}$ of $21=7\frac{1}{2}$ dollars, share of 1st, and $\frac{9}{14}$ of $21=13\frac{1}{2}$ dollars, share of 2nd.

(8) $\frac{60}{100}=\frac{3}{5}$; $\frac{3}{5}$ of $25=15$ dollars. *Ans.*

(9) $\frac{15}{100}=\frac{3}{20}$; $\frac{3}{20}$ of $80=12$ dollars. *Ans.*

(10) 180 sheep eat as much as 18 oxen. $27+18=45$. $\frac{27}{45}=\frac{3}{5}$. $\frac{3}{5}$ of $25=15$ dollars, what A pays. $\frac{18}{45}=\frac{2}{5}$, and $\frac{2}{5}$ of $25=10$ dollars, what B pays.

(11) 15 cows are 5×3 cows, and 5×2 horses are 10 horses; 10 horses eat as much as 15 cows. $14+10=24$. $\frac{14}{24}=\frac{7}{12}$, and $\frac{10}{24}=\frac{5}{12}$; $\frac{1}{12}$ of $60=5$, $\frac{7}{12}$ of $60=35$, and $\frac{5}{12}$ of $60=25$. Hence A should pay \$35, and B \$25.

(12) 15 oxen are 5×3 oxen, and 5×2 horses are 10 horses; hence 10 horses eat as much as 15 oxen. Since 1 horse eat as much as 20 sheep, 6 horses eat as much as 120 sheep; hence 15 oxen and 120 sheep eat as much as 16 horses. $8+16=24$, $\frac{8}{24}=\frac{1}{3}$, $\frac{16}{24}=\frac{2}{3}$; $\frac{1}{3}$ of $75=\$25$, A should pay; and $\frac{2}{3}$ of $75=\$50$, B should pay.

(14) \$50 for 4 months is the same as \$200 for 1 month, and \$60 for 5 months is the same as \$300 for 1 month. $200+300=500$; $\frac{200}{500}=\frac{2}{5}$, A's share, and $\frac{300}{500}=\frac{3}{5}$, B's share. $\frac{2}{5}$ of \$45=\$18, A's gain, and $\frac{3}{5}$ of \$45=\$27, B's gain.

(15) 3 men for 4 days=1 man for 12 days; and 5 men for 3 days=1 man for 15 days. $12+15=27$; $\frac{12}{27}=\frac{4}{9}$, $\frac{15}{27}=\frac{5}{9}$; $\frac{4}{9}$ of \$81=\$36, and $\frac{5}{9}$ of \$81=\$45.

(16) \$2 for 5 months is the same as \$10 for 1 month, and \$3 for 4 months is the same as \$12 for 1 month. $10+12=22$, $\frac{10}{22}=\frac{5}{11}$, $\frac{12}{22}=\frac{6}{11}$; $\frac{5}{11}$ of \$55=\$25, A's share, and $\frac{6}{11}$ of \$55=\$30, B's share.

(17) 4 horses for 5 mon. is the same as 1 horse for 20 mon., and 10 cows for 6 mon., as 1 cow for 60 mon.; but 1 cow for 60 mon. is the same as 3 cows for 20 mon., and 3 cows eat as much as 2 horses, and 2 horses for 20 mon. is the same as 1 horse for 40 mon. $20+40=60$, $\frac{20}{60}=\frac{1}{3}$, and $\frac{40}{60}=\frac{2}{3}$. $\frac{1}{3}$ of \$27=\$9, what E should pay; and $\frac{2}{3}$ of \$27=\$18, what F should pay.

LESSON IV.

(2) Of each 7 cents John gets 3, and James 4. In 28 cents there are 4 times 7 cents, hence John gets 4 times 3 cents which are 12 cents, and James 4 times 4 cents which are 16 cents.

(3) $4+3+2=9$. 9 in 45 is contained 5 times. $5\times4=20$, A's share. $5\times3=15$, B's share, and $5\times2=10$, C's share.

(4) $5+3=8$. 8 in 96 is contained 12 times; 12 times 5=60, apple trees; and 12 times 3=36, peach trees.

(5) Since there are 2 sheep for each cow, there are 6 sheep for each 3 cows, or each horse. $1+3+6=10$, that is, for each 10 animals there are 1 horse, 3 cows, and 6 sheep. 10 in 60 is contained 6 times; 6 times 1=6, the number of horses; 6 times 3=18, the number of cows; and 6 times 6=36, the number of sheep.

(6) $2+3=5$. 5 in 35 is contained 7 times; $7\times2=14$, the number of boys, and $7\times3=21$, the number of girls.

(7) The number and 3 times the number, that is 4 times the number, is 48, hence *once* the number is $\frac{1}{4}$ of 48, which is 12.

(8) $1+2+3=6$. 6 in 42 is contained 7 times; 7 times $1=7$, A's plums; 7 times $2=14$, B's plums; and 7 times $3=21$, C's plums.

(9) $1+4=5$; $\frac{1}{5}$ of $25=5$; $\frac{4}{5}$ of $25=20$. *Ans.* 5 yd. and 20 yd.

(10) Since Agnes gets twice as many as Emma, and Sarah twice as many as Agnes, Sarah gets 4 times as many as Emma. But $1+2+4=7$; hence of each 7 cherries Emma gets 1, Agnes 2, and Sarah 4. 7 in 35 is contained 5 times; $5\times1=5$, $5\times2=10$, and $5\times4=20$; hence Emma gets 5, Agnes 10, and Sarah 20 cherries.

(11) $3+4=7$: $\frac{1}{7}$ of 28 is 4; 3 times $4=12$; 4 times $4=16$.

LESSON V.

(12) $2+3=5$; $\frac{2}{5}$ of 25 cts.$=10$ cts., John's; $\frac{3}{5}$ of $25=15$ cts., George's.

(13) $5+7=12$; $\frac{5}{12}$ of $48=20$, the 1st. $\frac{7}{12}$ of $48=28$, the 2d.

(14) $3+4+5=12$; $\frac{3}{12}$ of $60=15$, $\frac{4}{12}$ of $60=20$, and $\frac{5}{12}$ of $60=25$.

(15) $1+2+3+4=10$; $\frac{1}{10}$ of $70=7$, $\frac{2}{10}$ of $70=14$, $\frac{3}{10}$ of $70=21$, and $\frac{4}{10}$ of $70=28$.

(17) $1\frac{1}{2}+2\frac{1}{2}=4$. 4 in 16 is contained 4 times; 4 times $1\frac{1}{2}=6$, Thomas' share; and 4 times $2\frac{1}{2}=10$, Moses' share.

(18) $\frac{1}{2}+\frac{1}{3}+\frac{1}{4}=\frac{13}{12}$. $\frac{13}{12}$ in 39 is contained 36 times; $36\times\frac{1}{2}=18$, $36\times\frac{1}{3}=12$, and $36\times\frac{1}{4}=9$. *Ans.* 18, 12, and 9.

(20) $1+1\frac{3}{4}=\frac{11}{4}$. $\frac{11}{4}$ in 33 is contained 12 times; 12 times $1=12$, and 12 times $1\frac{3}{4}=21$. *Ans.* John 12, James 21.

(21) $1+1\frac{2}{3}=\frac{8}{3}$. $\frac{8}{3}$ in 32 is contained 12 times; $12\times1=12$, age of Frank, and $12\times1\frac{2}{3}=20$, age of William.

(22) $1+2\frac{1}{3}=3\frac{1}{3}$. $3\frac{1}{3}$ in 30 is contained 9 times; 9 times $1=9$, the number of unsound, and 9 times $2\frac{1}{3}=21$, the number of sound apples.

(23) $1+\frac{4}{5}=\frac{9}{5}$. $\frac{9}{5}$ in 27 is contained 15 times; 15 times $1=15$, the number of feet the 2d can build, and 15 times $\frac{4}{5}=12$, the number the 1st can build.

(24) Since B has $\frac{1}{2}$ as many as A, and C $\frac{1}{2}$ as many as B, therefore C has $\frac{1}{2}$ of $\frac{1}{2}$, or $\frac{1}{4}$ as many as A. $1+\frac{1}{2}+\frac{1}{4}=\frac{7}{4}$. $\frac{7}{4}$ in 42 is contained 24 times; 24 times $1=24$, A's share; 24 times $\frac{1}{2}=12$, B's share; and 24 times $\frac{1}{4}=6$, C's share.

(25) Since there are $\frac{2}{3}$ as many sheep as hogs, and $\frac{3}{4}$ as many cows as sheep, for each hog there is $\frac{2}{3}$ of a sheep, and $\frac{3}{4}$ of $\frac{2}{3}=\frac{6}{12}=\frac{1}{2}$ of a cow. $1+\frac{2}{3}+\frac{1}{2}=\frac{13}{6}$, $\frac{13}{6}$ in $104=48$; $48\times1=48$, the number of hogs; $48\times\frac{2}{3}=32$, the number of sheep; and $48\times\frac{1}{2}=24$, the number of cows.

Lesson VI.

(2) $19-5=14$; $\frac{1}{2}$ of $14=7$, the number of apples James has; and $7+5=12$, the number Thomas has.

(3) $31-7=24$; $\frac{1}{2}$ of $24=12$, the less number; and $12+7=19$, the greater number.

(4) $32-8=24$; $\frac{1}{2}$ of $24=12$, the number each had at first.

(5) $20+4+6=30$; $\frac{1}{2}$ of $30=15$; $15-4=11$, the number Thomas had left; and $15-6=9$, the number William had left.

(6) $24+7+5=36$. Since Mary bought twice as many as Sarah, they both bought 3 times as many as Sarah, therefore Sarah bought $\frac{1}{3}$ of $36=12$ cherries, and Mary twice 12, or 24 cherries. $24-7=17$, the number of cherries Mary had left; and $12-5=7$, the number Sarah had left.

(7) $50-5=45$, and $\frac{1}{3}$ of $45=15$, the required number.

(8) $31-10=21$, which is $\frac{3}{4}$ of the number; hence $\frac{1}{3}$ of 21, or 7, is $\frac{1}{4}$ of the number, and $\frac{4}{4}$ is 4 times 7, or 28.

(9) $21+7=28$, $\frac{1}{4}$ of $28=7$ is $\frac{1}{5}$, and $\frac{5}{5}=35$, the number.

(10) $4+3=7$. $37-7=30$. $\frac{1}{3}$ of $30=10$, Henry's age; $10+3=13$, Oliver's age; and $10+4=14$, James' age.

(11) Since Sarah has 3 cents less than Mary, she has only 5 more than Jane. $8+5=13$, $43-13=30$, $\frac{1}{3}$ of $30=10$, Jane's; $10+8=18$, Mary's; $18-3=15$, Sarah's.

(12) Twice the age of Frank, plus his age, less 3 years, is 42 years; hence 3 times the age of Frank must be 45 years. $\frac{1}{3}$ of 45=15 years, the age of Frank, and twice 15, less 3 years, is 27 years, the age of Mary.

(13) The ring cost \$5, and the watch \$12 more than the chain; hence 3 times the cost of the chain, and \$17 is equal to \$62; therefore 3 times the cost of the chain is equal to \$45. $\frac{1}{3}$ of 45=\$15, the cost of the chain; \$15+5=\$20, the cost of the ring; and \$15+12=\$27, the cost of the watch.

(14) 30+6=36, which is $\frac{2}{7}$ of his money. $\frac{1}{2}$ of 36=18, which is $\frac{1}{7}$ of his money, and 7 times 18=126 cents, his money.

(15) The money of John and James=3 times the money of James and \$3; hence the money of Frank=3 times the money of James and \$10; and the money of all three, \$55,=6 times the money of James and \$13, hence 6 times the money of James is \$42. $\frac{1}{6}$ of 42=\$7, James' money; 7+7+3=\$17, John's, and 7+17+7=\$31, Frank's.

LESSON VII.

(2) $\frac{7}{7}+\frac{3}{7}=\frac{10}{7}$. $\frac{1}{10}$ of 60=6, which is $\frac{1}{7}$. $\frac{7}{7}$=42, and $\frac{3}{7}$=18; hence Thomas pays \$42, and John \$18.

(3) From noon to midnight is 12 hours. Since the time past noon=$\frac{1}{2}$ the time to midnight, therefore the time and $\frac{1}{2}$ the time, or $\frac{3}{2}$ of the time to midnight=the time from noon to midnight, 12 hours. If $\frac{3}{2}$ is 12 hours, $\frac{1}{2}$ is $\frac{1}{3}$ of 12 hours, or 4 hours; that is, one-half the time to midnight, which is the same as the time past noon, is 4 hours. Hence the time was 4 o'clock, afternoon.

(4) $1+\frac{3}{5}=\frac{8}{5}$, $\frac{1}{8}$ of 12 hours=$1\frac{1}{2}$ hours, which is $\frac{1}{5}$ and $\frac{3}{5}=4\frac{1}{2}$ hours; hence the hour was half past 4 P. M.

(5) $1+\frac{3}{4}=\frac{7}{4}$. $\frac{1}{7}$ of 56=8=$\frac{1}{4}$; $\frac{4}{4}$=32, and $\frac{3}{4}$=24, the distance traveled each day respectively.

(6) Since the first, plus $\frac{5}{7}$ of the first less 8, that is $\frac{12}{7}$ of the first less 8=100, $\frac{12}{7}$ of the first is 108. $\frac{1}{12}$ of 108=9, is $\frac{1}{7}$ of the first part. $\frac{7}{7}$=63, the 1st; $\frac{5}{7}$=45, and 45—8=37, the 2nd part.

(7) The first$+\frac{1}{2}+\frac{3}{4}$, that is $\frac{9}{4}$ of the first$=45$. $\frac{1}{9}$ of $45=5$, $\frac{1}{4}$ of the first. $\frac{4}{4}=20$, the first part; $\frac{1}{2}$ of $20=10$, the second; and $\frac{3}{4}$ of $20=15$, the third.

(8) $\frac{2}{3}$ of $\frac{3}{5}$=twice $\frac{1}{3}$ of $\frac{3}{5}=\frac{2}{5}$; hence C has $\frac{2}{5}$ as many as A. $1+\frac{3}{5}+\frac{2}{5}=\frac{10}{5}$; $\frac{1}{10}$ of $40=4$, which is $\frac{1}{5}$. $\frac{5}{5}=20$, A's; $\frac{3}{5}=12$, B's; and $\frac{2}{5}=8$, C's.

(9) $\frac{3}{5}$ of $\frac{5}{6}=3$ times $\frac{1}{5}$ of $\frac{5}{6}=\frac{3}{6}=\frac{1}{2}$. $1+\frac{5}{6}+\frac{1}{2}=\frac{6}{6}+\frac{5}{6}+\frac{3}{6}=\frac{14}{6}$ hence $\frac{14}{6}$ of the top part$=70$; $\frac{1}{14}$ of $70=5$, which is $\frac{1}{6}$; $\frac{6}{6}=30$, top; $\frac{5}{6}=25$, middle; and $\frac{3}{6}=15$, the lower part.

(10) $\frac{3}{4}$ of $\frac{2}{5}=\frac{6}{20}=\frac{3}{10}$. $1+\frac{2}{5}+\frac{3}{10}=\frac{10}{10}+\frac{4}{10}+\frac{3}{10}=\frac{17}{10}$; hence $\frac{17}{10}$ of the cost was $\$34$. $\frac{1}{17}$ of 34 is 2, which is $\frac{1}{10}$; $\frac{10}{10}=\$20$, the cost of the coat; $\frac{4}{10}=\$8$, the cost of the hat; and $\frac{3}{10}=\$6$, the cost of the vest.

(11) If $\frac{1}{2}$ the cows$=\frac{2}{7}$ the sheep, the cows$=\frac{4}{7}$ of the sheep. $1+\frac{4}{7}=\frac{11}{7}$; hence $\frac{11}{7}$ of the sheep is 55. $\frac{1}{11}$ of $55=5$, which is $\frac{1}{7}$; $\frac{7}{7}=35$; the sheep; and $\frac{4}{7}=20$, the cows.

(12) If $\frac{1}{3}$ of the less$=\frac{2}{9}$ of the greater, the $\frac{3}{3}$, or the whole of the less$=3\times\frac{2}{9}$, which are $\frac{2}{3}$ of the greater. $1+\frac{2}{3}=\frac{5}{3}$; hence $\frac{5}{3}$ of the greater$=100$; $\frac{1}{5}$ of $100=20$ is $\frac{1}{3}$ of the greater. $\frac{3}{3}=60$, the greater number. $\frac{2}{3}=40$, the less.

(13) If $\frac{1}{4}$ of Mary's age$=\frac{1}{3}$ of Sarah's, $\frac{4}{4}$ of Mary's$=\frac{4}{3}$ of Sarah's. $1+\frac{4}{3}=\frac{7}{3}$. $\frac{1}{7}$ of $14=2$, is $\frac{1}{3}$ of Sarah's. $\frac{3}{3}=6$, Sarah's age; and $\frac{4}{3}=8$, Mary's age.

(15) If $\frac{2}{3}$ of the first$=\frac{3}{4}$ of the second, $\frac{1}{3}=\frac{1}{2}$ of $\frac{3}{4}$, or $\frac{3}{8}$ of the second, and $\frac{3}{3}$, or the whole of the first$=\frac{9}{8}$ of the second. Hence the second plus $\frac{9}{8}$ of the second, or $\frac{17}{8}$ of the second$=51$. $\frac{1}{17}$ of $51=3$ is $\frac{1}{8}$ of the second. $\frac{8}{8}=24$, the second part; and $\frac{9}{8}=27$, the first part.

(16) If $\frac{2}{3}$ of the apple trees$=\frac{4}{7}$ of the peach trees, $\frac{1}{3}=\frac{2}{7}$, and $\frac{3}{3}$, or all the apple trees$=\frac{6}{7}$ of the peach trees. Hence the peach trees and $\frac{6}{7}$ of the peach trees, that is, $\frac{13}{7}=65$. $\frac{1}{13}$ of $65=5$, is $\frac{1}{7}$. $\frac{7}{7}=35$, the peach trees; and $\frac{6}{7}=30$, the apple trees.

(17) Since $\frac{2}{3}$ A's distance$=\frac{5}{9}$ B's, $\frac{1}{3}$ A's$=\frac{1}{2}$ of $\frac{5}{9}$, or $\frac{5}{18}$ of B's, and $\frac{3}{3}$ of A's$=3\times\frac{5}{18}$, or $\frac{5}{6}$ of B's; hence B's and $\frac{5}{6}$ of B's, that is $\frac{11}{6}$, is 66 miles. $\frac{1}{11}$ of 66 is 6 miles, $\frac{1}{6}$ of

B's, and since the difference between B's and $\frac{5}{6}$ of B's is $\frac{1}{6}$ of B's, B traveled 6 miles further than A.

(18) Since once the time past noon+3 hours is $\frac{1}{2}$ the time to midnight, twice the time past noon+6 hours=the whole time to midnight; but the time past noon+the time to midnight is 12 hr., hence the time past noon, with twice the time past noon + 6 hr. = 12 hr.; hence 3 times the time past noon is 6 hours, and the time past noon is $\frac{1}{3}$ of 6 hr., or 2 hr.

(19) See the "Explanation." If 12 hours is $\frac{4}{5}$, $\frac{1}{4}$ of 12, which is 3, is $\frac{1}{5}$; hence the hour is 3 in the afternoon.

(20) Here 12 hours must be $\frac{3}{4}$ of the whole time; hence $\frac{1}{4}$ must be $\frac{1}{3}$ of 12 hours, which is 4 hours; that is, the hour is 4 P. M.

(21) If $\frac{1}{2}$ the time past noon is $\frac{1}{20}$ of the time past midnight, the time past noon must be $\frac{1}{10}$ of the time past midnight; then the time past noon is $\frac{1}{10}$ of the whole time from midnight, and the time from midnight to noon must be $\frac{9}{10}$ of the whole time. $\frac{1}{9}$ of 12 hours is $1\frac{1}{3}$ hours, that is, the hour is $1\frac{1}{3}$ P. M., or 20 minutes past 1 o'clock.

Lesson VIII.

(2) $1+\frac{2}{3}=\frac{5}{3}$. $\frac{1}{5}$ of 20=4, is $\frac{1}{3}$ of the number; $\frac{3}{3}$=12, the number.

(3) $1+\frac{2}{5}=\frac{7}{5}$. $\frac{1}{7}$ of 21=3, is $\frac{1}{5}$ of the age; $\frac{5}{5}$=15, her age.

(4) Twice the number and $\frac{3}{5}$ of the number equals $\frac{13}{5}$ of it. $\frac{1}{13}$ of 52 is 4, which is $\frac{1}{5}$; $\frac{5}{5}$=20, the number.

(5) Twice the number less its $\frac{4}{7}$, is $\frac{14}{7}$ less $\frac{4}{7}$, or $\frac{10}{7}$; if $\frac{10}{7}$ is 40, $\frac{1}{10}$ of 40, which is 4, is $\frac{1}{7}$, and $\frac{7}{7}$, or the number is 28.

(6) Three times the number less its $\frac{3}{5}$ is $\frac{15}{5}$ less $\frac{3}{5}$, or $\frac{12}{5}$; if $\frac{12}{5}$ is 48, $\frac{1}{12}$ of 48, 4, is $\frac{1}{5}$, and $\frac{5}{5}$ is 20.

(7) $1+\frac{1}{2}+\frac{2}{3}=\frac{6}{6}+\frac{3}{6}+\frac{4}{6}=\frac{13}{6}$. $\frac{1}{13}$ of 26=2, is $\frac{1}{6}$, and $\frac{6}{6}$=12, his age.

(8) Her age, $\frac{1}{3}$ and $\frac{1}{4}$ of her age are $\frac{12}{12}+\frac{4}{12}+\frac{3}{12}$, or $\frac{19}{12}$; but twice her age is $\frac{24}{12}$; hence $\frac{19}{12}$ of her age plus 10 years $=\frac{24}{12}$; $\frac{5}{12}$ of her age is 10 years, and her age, 24 years.

(9) $1-\frac{2}{5}=\frac{3}{5}$. $\frac{3}{5}$ is 30 cents; hence $\frac{5}{5}$ is 50 cents.

(10) $1+\frac{1}{2}+\frac{3}{5}=\frac{10}{10}+\frac{5}{10}+\frac{6}{10}=\frac{21}{10}$; treble the number is $\frac{30}{10}$, hence $\frac{9}{10}$ of the number is 27, and the number is 30.

(11) $1-\frac{3}{11}=\frac{8}{11}$. $\frac{1}{8}$ of $40=5$, is $\frac{1}{11}$ of the father's age, or 55 years; hence the son's=15 years.

(12) $1+\frac{4}{5}=\frac{9}{5}$. 3 times her age is $\frac{15}{5}$. Hence $\frac{6}{5}$ of her age is 18 years, and her age is 15 years.

(13) $1-\frac{2}{9}=\frac{7}{9}$. $\frac{1}{7}$ of $28=4$ yards is $\frac{1}{9}$; $\frac{9}{9}=36$ yards. *Ans.*

(14) Twice the distance from A to B is equal to the distance from C to D; hence twice the distance from A to B is equal to $\frac{2}{3}$ of the same distance plus 20 miles; therefore 20 miles is $\frac{4}{3}$ of the distance from A to B, which is 15 miles, and from C to D is 30 miles.

(15) $1+\frac{1}{3}+\frac{1}{5}+\frac{5}{9}=\frac{45}{45}+\frac{15}{45}+\frac{9}{45}+\frac{25}{45}=\frac{94}{45}$; $\frac{1}{94}$ of 94 is 1, which is $\frac{1}{45}$; hence the age is 45 years.

LESSON IX.

(1) *Ans.* $\frac{1}{2}$. **(2)** *Ans.* $\frac{1}{4}$. **(3)** *Ans.* 2.

(4) *Ans.* as many as $\frac{3}{8}$ of a day is contained times in 1 day, that is, $2\frac{2}{3}$.

(6) He could dig 2 trenches in 7 da., or $\frac{2}{7}$ of 1 trench in 1 da.

(7) He could walk 5 times the distance in 16 days, or $\frac{5}{16}$ of the distance in 1 day: and in 2 days $\frac{10}{16}=\frac{5}{8}$.

(8) *Ans.* $\frac{1}{2}+\frac{1}{4}$, which are $\frac{3}{4}$.

(9) *Ans.* $\frac{1}{2}+\frac{1}{4}+\frac{1}{5}$, which are $\frac{19}{20}$.

(11) $\frac{1}{6}+\frac{1}{12}=\frac{3}{12}=\frac{1}{4}$; $\frac{1}{4}$ in 1 is contained 4 times. *Ans.* 4 da.

(12) $\frac{1}{5}+\frac{1}{7}=\frac{12}{35}$; $\frac{12}{35}$ in 1, $2\frac{11}{12}$ times. *Ans.* $2\frac{11}{12}$ days.

(13) $\frac{1}{2}+\frac{1}{3}+\frac{1}{6}=\frac{6}{6}=1$; hence all three can do it in 1 day.

(14) $\frac{1}{3}+\frac{1}{5}+\frac{1}{6}=\frac{21}{30}=\frac{7}{10}$; $\frac{7}{10}$ in 1, $1\frac{3}{7}$ times. *Ans.* $1\frac{3}{7}$ hr.

(16) $\frac{1}{12}-\frac{1}{30}=\frac{1}{20}$; $\frac{1}{20}$ in 1 is contained 20 times. *Ans.* 20 da.

(17) A and B can do $\frac{1}{8}+\frac{1}{12}=\frac{5}{24}$ in 1 da. $\frac{1}{4}-\frac{5}{24}=\frac{1}{24}$, the part that C can do in a da.; hence he can do it in 24 da.

(19) The first will empty the cistern 3 times in 1 hour, and the second 5 times in 1 hour; hence both will empty it 8 times in 1 hour, or once in $\frac{1}{8}$ hour.

(20) $\frac{7}{10}$ days in 1 day is contained $1\frac{3}{7}$ times. *Ans.* $1\frac{3}{7}$ days.

(21) C can reap $\frac{1}{5}$ of the field in 1 day, and since B can reap it in $3\frac{1}{3}$ days, he can reap 3 such fields in 10 days, or $\frac{3}{10}$ of a field in 1 day. $\frac{1}{5}+\frac{3}{10}=\frac{5}{10}=\frac{1}{2}$; hence both can reap $\frac{1}{2}$ the field in 1 day, or the whole field in 2 days.

(22) A can do $\frac{1}{3}$ of his piece in 1 da. If B can do a piece 3 times as large in 7 da., he can do $\frac{3}{7}$ of such a piece in 1 da.; hence both can do $\frac{1}{3}+\frac{3}{7}=\frac{16}{21}$ of it in 1 da., or the whole in as many da. as $\frac{16}{21}$ is contained times in 1. $\frac{16}{21}$ in 1, $1\frac{5}{16}$ times. To do a piece of work 5 times as large will require 5 times as long. $5\times1\frac{5}{16}$ da.$=6\frac{9}{16}$ da. *Ans.*

(23) $\frac{1}{20}$ of 100 gallons$=$5 gallons to be emptied per minute: the 1st and 2d empty 2 gallons in a minute; hence the third must empty 3 gallons per minute, and in 20 minutes 60 gallons.

Or, the 1st and 2d empty 40 gallons in 20 minutes and the 3d must empty 100—40=60 gallons.

LESSON X.

(2) *Ans.* $3\frac{3}{8}$. **(3)** *Ans.* $5\frac{1}{2}$. **(4)** *Ans.* $5\frac{5}{9}$. **(5)** *Ans.* $1\frac{3}{5}$.

(6) *Ans.* $\frac{7}{12}$. **(7)** *Ans.* $2\frac{2}{15}$.

(8) *Ans.* 11. **(9)** *Ans.* 20 Thus, if she gave away $\frac{4}{9}$ of what she received, she had $\frac{5}{9}$ left; $\frac{5}{9}$ of $\frac{3}{5}=\frac{1}{3}$, and $\frac{1}{3}$ of 60 is 20.

(10) Since $\frac{2}{7}$ remains, 35 is $\frac{5}{7}$, and $\frac{2}{7}$ is 14 miles. $\frac{3}{7}$ of 14 is 6, and 14 less 6 is 8 miles, the required distance.

(11) *Ans.* 25. **(12)** $\frac{4}{9}$ of 27 is 12, $\frac{3}{4}$ of 8 is 6; 12 and 6 are 18, which is $\frac{3}{5}$ of 30. *Ans.* $30.

(13) After losing $\frac{2}{7}$ she had $\frac{5}{7}$ left, and after giving $\frac{2}{5}$ of $\frac{5}{7}$ to Sarah, she had $\frac{3}{5}$ of $\frac{5}{7}$ left. $\frac{3}{5}$ of $\frac{5}{7}=\frac{3}{7}$. If $\frac{3}{7}$ is 6, $\frac{7}{7}$ is 14. *Ans.*

(14) *Ans.* 20 cents. **(15)** $\frac{5}{4}$ of 36 is 45, and 45 is $\frac{9}{7}$ of 35. *Ans.*

(16) If $\frac{1}{2}$ of B's$=\frac{2}{7}$ of A's, then all of B's$=\frac{4}{7}$ of A's : $1-\frac{4}{7}=\frac{3}{7}$ of A's$=12$: $\frac{1}{7}=4$: $\frac{7}{7}=28$ cts., A's; and $\frac{4}{7}=16$ cts., B's.

(17) $\frac{1}{3}+\frac{1}{4}+\frac{1}{12}=\frac{8}{12}=\frac{2}{3}$; hence 32 is $\frac{1}{3}$, and the whole is 96, of which 32 are apple trees, 24 pear trees, and 8 plum trees.

(18) The whole less $\frac{2}{9}$ is $\frac{7}{9}$, and the difference between $\frac{7}{9}$ and $\frac{2}{9}$ is $\frac{5}{9}$; if $\frac{5}{9}$ are 25, $\frac{1}{9}$ is 5, and $\frac{2}{9}$ are 10, the pear trees, and $\frac{7}{9}$ are 35, the apple trees.

(19) Since the plum trees are $\frac{1}{3}$ of the apple trees, $\frac{1}{4}$ of the plum trees$=\frac{1}{12}$ of the apple trees; hence the cherry trees $=\frac{1}{2}$ and $\frac{1}{12}$, that is, $\frac{7}{12}$ of the apple trees; and the whole number of trees=once the apple trees, $\frac{1}{3}$ of the apple trees, and $\frac{7}{12}$ of the apple trees; that is, $1+\frac{1}{3}+\frac{7}{12}$, or $\frac{23}{12}$, of the apple trees=69. $\frac{1}{23}$ of 69 is 3, which is $\frac{1}{12}$. $\frac{12}{12}=36$, the apple trees; $\frac{1}{3}=12$, the plum trees; and $\frac{7}{12}=21$, the cherry trees.

(20) $\frac{5}{3}$ of 12 years is 20 years, which is $\frac{4}{9}$ of the ages of Jane and Sarah; if 20, is $\frac{4}{9}$, $\frac{1}{4}$ of 20, or 5, is $\frac{1}{9}$, and $\frac{9}{9}=45$ years, the sum of their ages. Then Sarah's age and $\frac{7}{8}$ of her age, or $\frac{15}{8}$ of her age, are 45 years; $\frac{1}{15}$ of 45 is 3, which is $\frac{1}{8}$ of her age, and $\frac{8}{8}$, or her age, is 24 years; $\frac{7}{8}$ of 24 years is 21 years, Jane's age.

(21) $\frac{3}{11}$ of $55=15$; $\frac{4}{5}$ of 30, or 24, is $\frac{4}{9}$ of 54: twice $54=108$; 15 in 108, $7\frac{1}{5}$ times. *Ans.*

(22) $\frac{3}{4}$ of John's money$=\frac{3}{4}$ of $\frac{3}{5}$ of Charles'$=\frac{9}{20}$: $\frac{9}{20}$ of Charles'+\$33=Charles' money; hence \$33 is $\frac{11}{20}$, and Charles has \$60, John has $\frac{3}{5}$ of \$60=\$36.

LESSON XI.

(2) Since 4 of John's steps are equal to 5 of Henry's, 1 of John's is equal to $1\frac{1}{4}$ of Henry's, and 5 of John's to $6\frac{1}{4}$ of Henry's; but while John takes 5 steps, Henry takes 6. Hence John takes 5 steps to gain $\frac{1}{4}$ of one of Henry's steps on him, and therefore must take 20 steps to gain 1 of Henry's steps on him.

(3) In taking 7 steps, John gains 2 steps on Henry. Hence, to gain 30 steps, he must take as many times 7 steps as 2 steps is contained times in 30 steps. 2 in 30 is contained 15 times; 15 times 7 steps are 105 steps. *Ans.*

(5) Since 3 leaps of the hound are equal to 10 of the hare, and the hound takes 3 leaps while the hare takes 5; therefore, in taking 3 leaps, the hound gains 5 of the hare's leaps on her; and to gain 100 leaps, he must take as many times 3 leaps as 5 leaps are contained times in 100 leaps. 5 in 100 is contained 20 times, and 20 times 3 leaps are 60 leaps. *Ans.*

(6) Since 4 of M's steps=7 of N's, 1 of M's=$1\frac{3}{4}$ of N's, and 5 of M's =$8\frac{3}{4}$ of N's; hence while M takes 5 steps he gains $1\frac{3}{4}$ of N's steps on him, and to gain 35 steps, he must take as many times 5 steps as $1\frac{3}{4}$ steps are contained times in 35 steps. $1\frac{3}{4}$ in 35, are contained 20 times, and 20×5 steps=100 steps. *Ans.*

(7) Since 2 of the hound's leaps=3 of the hare's, and the hound takes 2 leaps while the hare takes 3, the hound goes just as fast as the hare, and will not catch it, but will continue at the distance of 8 leaps from it.

(9) Since the tail is as long as the head and $\frac{1}{3}$ of the body, therefore the tail is 4 inches plus $\frac{1}{3}$ the body, and the body is equal to 4 inches, plus 4 inches, plus $\frac{1}{3}$ of the body; hence $\frac{2}{3}$ of the body is 8 inches, and $\frac{1}{3}$ is 4 inches. $\frac{3}{3}$=12 inches, the length of the body; 4+4=8 inches, the tail; and 12+8+4=24 inches, the whole length.

(10) Since C has as many as A+$\frac{2}{3}$ as many as B, C has 10 cents+$\frac{2}{3}$ of B's; but B's money=A's and C's; hence B's=10 cents+10 cents+$\frac{2}{3}$ of B's; or $\frac{1}{3}$ of B's=20 cents; hence B has 60 cents, and C 50 cents.

(11) Since the cow cost as much as the sheep and $\frac{1}{4}$ of the horse, therefore the cow cost 8 dollars and $\frac{1}{4}$ of the horse; but the horse cost twice as much as both the sheep and cow; hence the cost of the horse was equal to 16 dollars, plus 16 dollars, plus $\frac{1}{2}$ the cost of the horse; therefore $\frac{1}{2}$ the cost of the horse was 32 dollars. Hence the horse cost $64, and the cow $24.

(12) Since the tail weighs 3 pounds more than the head and $\frac{1}{2}$ the body, therefore the tail weighs 11 pounds plus $\frac{1}{2}$ the body; but the body weighs as much as the head and tail; hence the body is equal to 8 pounds plus 11 pounds plus $\frac{1}{2}$ the body; therefore $\frac{1}{2}$ the body weighs 19 pounds, and the whole 38 pounds. The tail weighs $11+19=30$ pounds, and the whole fish 76 pounds.

(13) If 2 of B's steps=3 of A's, then 4 of B's=6 of A's: hence, B gains 1 of A's in moving 6 of A's, that is, $\frac{1}{6}$ of the distance B travels; $\frac{1}{6}$ of 30 ft.=5 ft. *Ans.*

Lesson XII.

(2) The difference between 4 peaches and 2 peaches is 2 peaches, and the difference between having 9 left, or 3 left, is 6; hence giving 2 peaches more to each would increase the number given by 6. Hence he has as many children as 2 peaches is contained times in 6 peaches. 2 in 6 is contained 3 times. *Ans.* 3 children. 3 times $2=6$, and $6+9=15$, the number of peaches he had.

(3) The difference between giving 5 and 8 to each is 3, and between having none left and 21 left is 21; $21\div3=7$; hence she had 7 playmates, and 56 cherries.

(4) The difference between paying 1 dollar and $1\frac{1}{2}$ dollars a yard is $\frac{1}{2}$ a dollar, and the difference between having 5 dollars left and nothing left is 5 dollars. Hence she wished to buy as many yards as $\frac{1}{2}$ dollar is contained times in 5 dollars, which is 10 times; therefore she wished to buy 10 yards of silk, and had 15 dollars.

(5) The difference between 29 over and 6 lacking is 35 cts.; 8 cts.—3 cts.=5 cts. 5 cts. in 35 cts., 7 times. $7\times8=56$, and $56-6=50$ cts. *Ans.*

(6) $12\frac{1}{2}-10=2\frac{1}{2}$; $2\frac{1}{2}$ in 40, 16 times. *Ans.* 16 yards.

(8) 3 times the age of James less his age is twice his age, or 10 years; hence his age is $\frac{1}{2}$ of 10 years, or 5 years; and Thomas' age is 15 years.

(9) Since B's age is twice C's, and A's 5 times B's, therefore A's is 10 times C's, and the difference between A's and C's is 9 times C's, which is 45 years. Hence C's age is 5 years; B's 10 years; and A's 50 years.

(10) Since A's money $=\frac{1}{2}$ B's, twice A's=B's; and since

B's money is $\frac{1}{3}$ C's, twice A's money is $\frac{1}{3}$ C's, and 6 times A's money=C's. But the difference between A's and C's money is \$15; therefore 6 times A's money less A's money is \$15. Hence 5 times A's money is \$15, and A's money is \$3. Therefore B's money is \$6, and C's \$18.

(11) Since the second contains 4 times as many as the first, and the third 3 times as many as the second, the third contains 12 times as many as the first. Both the first and second contain 5 times the first, and 12 times less 5 times the first, or 7 times the first is the excess of the third above both the first and second, which is 70 sheep; hence 7 times the first are 70 sheep, and the first contains 10 sheep, the second 40, and the third 120.

(12) If $\frac{1}{3}$ of C's age=twice A's, C's age must be 6 times A's; B's is twice A's; the difference between B's and C's will be 4 times A's=20 years; hence A's age is 5 years, and B's 10 years, C's 30 years.

(13) A daughter and a son together received 3 dollars, hence there were as many of each as 3 dollars is contained times in 18 dollars, which is 6; therefore he had 6 sons and 6 daughters, in all 12 children.

(15) If he had worked every day he would have received 9 dollars, but as he received only 5 dollars, he lost 4 dollars by being idle; he lost 50 cents each day he was idle, that is, his wages, 30 cents, and the 20 cents he paid for his board; hence he was as many days idle as 50 cents is contained times in 4 dollars, which is 8. *Ans.* 8 days idle, and therefore he worked 22 days.

(17) The difference between 4 yr. ago and 4 yr. hence is 8 yr.; 3 times B's age (which was A's) plus 8 yr. was equal to twice (B's age plus 8 yr.); that is, 3 times B's age plus 8 yr. was equal to twice B's age plus 16 yr., hence B's age was 8 yr., and A's 24 yr.

(18) In 10 years, A's age+10 years—$\frac{3}{5}$ of (twice A's+10 years)=$\frac{6}{5}$ of A's+6 years : hence $\frac{1}{5}$ of A's=10—6=4 : $\frac{5}{5}$=20 years, A's age, and 40 years=B's.

(20) Since the first horse and \$12 are worth $\frac{1}{3}$ the second; 3

times the first horse and \$36 are worth the second; but the second horse and \$12 are worth 4 times the first; hence 3 times the first horse and \$36, and \$12, are worth 4 times the first horse; hence the first is worth \$48; and the second \$180

(21) If $\frac{3}{2}$ cost $\frac{5}{2}$ pence, $\frac{1}{2}$ will cost $\frac{5}{6}$, and 1, $\frac{5}{3}$ penny : hence, for 9 pence, you can buy as many as $\frac{5}{3}$ are contained times in 9 or $\frac{27}{3}$, which is $5\frac{2}{5}$ *Ans.*

LESSON XIII.—MISCELLANEOUS EXAMPLES.

(1) *Ans.* 105. **(2)** *Ans.* \$60.

(3) If A walks 10 miles in $1\frac{1}{4}$ days, he walks 8 miles in 1 day, and if B walks 8 miles in $1\frac{3}{5}$ days, he walks 5 miles a day. It takes A $2\frac{1}{2}$ days to travel 20 miles; and in $2\frac{1}{2}$ days B will travel $12\frac{1}{2}$ miles.

(4) 2 apples for 3 cents and 2 apples for 5 cents are 4 apples for 8 cents, or 1 apple for 2 cents, which is the rate of buying. 3 apples for 7 cents is 1 apple for $2\frac{1}{3}$ cents, the rate of selling; hence on each apple sold he gained $\frac{1}{3}$ of a cent, and on each dozen, 12 times $\frac{1}{3}$ of a cent, which is 4 cents.

(5) For each egg at 2 cts. he bought 2 eggs at 3 cts. a piece; hence for each 3 eggs he paid 8 cts.$=2\frac{2}{3}$ cts. a piece. By selling them at 3 for 10 cts. he sold them at $3\frac{1}{3}$ cts. a piece; hence on each egg he gained $3\frac{1}{3}—2\frac{2}{3}$, or $\frac{2}{3}$ cts., and on each dozen 8 cts. Had he sold them at 4 for 10 cts., or $2\frac{1}{2}$ cts. a piece he would have lost $2\frac{2}{3}—2\frac{1}{2}$, or $\frac{1}{6}$ cts. a piece, or 2 cts. per dozen.

(6) 2 for 1 cent is $\frac{1}{2}$ cent a piece; 4 for 1 cent is $\frac{1}{4}$ cent a piece. Hence each 2 pears cost $\frac{1}{2}+\frac{1}{4}$, or $\frac{3}{4}$ cent, and each $\frac{3}{8}$ cent. He sold them at $\frac{3}{5}$ cent a piece; $\frac{3}{5}$ less $\frac{3}{8}$ $=\frac{9}{40}$; hence he gained $\frac{9}{40}$ of a cent on each pear sold; and to gain 18 cents he must sell as many pears as $\frac{9}{40}$ is contained times in 18 cents, which is 80. *Ans.* 80 pears.

(7) Each duck cost $\$\frac{1}{6}$, and each 2 chickens $\$\frac{1}{4}$; therefore 2 chickens and a duck together cost $\frac{1}{6}+\frac{1}{4}=\$\frac{5}{12}$. $\$\frac{1}{2}—\frac{5}{12}$

$=\$\frac{1}{12}$, what he gained on each lot. Hence he sold as many lots as $\frac{1}{12}$ is contained times in $\$2\frac{1}{2}$, which is 30; therefore he sold 30 ducks, and 60 chickens.

(8) $10\times4=40$, and $3\times40=120$, the hours work of 1 man; $\frac{1}{8}$ of $120=15$, the hours work of 8 men; and 15 hours$=2\frac{1}{2}$ days of 6 hours each. *Ans.* $2\frac{1}{2}$ days.

(9) Reserving 2 for Mary and 3 for Lucy, there are left 27 to be divided equally. $\frac{1}{3}$ of 27 is 9, James' share. $9+2=11$, Mary's share; and $9+3=12$, Lucy's share.

(10) Ten gal. in 1 hr. is 20 gal. in 2 hr., and $20-17=3$, the number of gal. left in each 2 hr.; hence it will require as many times 2 hr. to fill the vessel as 3 gal. is contained times in 15 gal. *Ans.* 10 hr.

(11) If A can do the work in $4\frac{1}{2}$ days, he can do twice the work in 9 days, or $\frac{2}{9}$ of the work in 1 day. If A and B together can do the work in $2\frac{4}{7}$ days, they can do 7 times the work in 18 days, or $\frac{7}{18}$ of the work in 1 day. $\frac{7}{18}-\frac{2}{9}=\frac{3}{18}=\frac{1}{6}$, the part of the work B can do in 1 day. Hence B can do the work in 6 days.

(12) A, B, and C can do $\frac{1}{5}$ of the work in 1 day; A and B do $\frac{1}{8}$ of it in 1 day; and B and C $\frac{1}{9}$ of it in 1 day. Hence $\frac{1}{5}-\frac{1}{8}=\frac{3}{40}$, the part C does in 1 day, therefore he can do it in $13\frac{1}{3}$ days. $\frac{1}{5}-\frac{1}{9}=\frac{4}{45}$, the part A does in 1 day, therefore he can do it in $11\frac{1}{4}$ days. $\frac{1}{9}-\frac{3}{40}=\frac{13}{360}$, the part B does in 1 day, therefore he can do it in $27\frac{9}{13}$ days.

(13) Since 5 men, *or* 7 women, can do the work in the same time, both together can do it in half the time of either; that is, $\frac{1}{2}$ of 35 days, or $17\frac{1}{2}$ days.

(14) Since 1 woman can do $\frac{3}{4}$ of a man's work, 4 women can do 3 men's work; hence the work of 2 men and 4 women =the work of 2 men and 3 men, 5 men; and the work of 1 man and 1 woman=the work of $1\frac{3}{4}$ men; hence it will take 1 man and 1 woman as many times 28 days as $1\frac{3}{4}$ is contained times in 5. $1\frac{3}{4}$ in $5=2\frac{6}{7}$ times, and $2\frac{6}{7}\times28$ days are 80 days. *Ans.*

(15) Both consumed $\frac{1}{15}$ in 1 day, and $\frac{6}{15}$ or $\frac{2}{5}$ in 6 days. After

$\frac{2}{5}$ were consumed $\frac{3}{5}$ were left, and if the woman consumed $\frac{3}{5}$ in 30 days, she consumed $\frac{1}{5}$ in 10 days, or the whole in 50 days. $\frac{1}{15}-\frac{1}{50}=\frac{7}{150}$, the part the man consumed in 1 day; hence it would last him $\frac{150}{7}=21\frac{3}{7}$ days.

(16) 36 at 3 for 4 cents cost 48 cents, 24 at 4 for 3 cents cost 18 cents. $36+24=60$, and $80-60=20$, which at 10 for 17 cents bring 34 cents. All the eggs at these rates bring $48+18+34=100$ cents. 80 eggs for 100 cents= $1\frac{1}{4}$ cent a piece, or 15 cents a dozen.

(17) James' money+$\frac{3}{4}$ of James' money+$\$6$, or $\frac{7}{4}$ James' money+$\$6=\$34$. Hence $\frac{7}{4}$ is $\$28$, and $\frac{1}{4}$ is $\$4$; $\frac{4}{4}=\$16$, James' money; and $\$18$=Thomas' money.

(18) Here A's money plus $\frac{2}{5}$ of A's money less $\$5$ is equal to $\$51$; or, $\frac{7}{5}$ of A's money is equal to $\$56$. Hence $\frac{1}{5}$ is $\frac{1}{7}$ of $\$56$, which is $\$8$, and $\frac{5}{5}$, or A's money, is $\$40$; and B's money is $\$11$.

(19) Since A's age is twice B's, therefore $2\frac{1}{2}$ times A's must be equal to 5 times B's age; but $\frac{3}{5}$ of B's age plus 44 years is equal to $2\frac{1}{2}$ times A's age; hence $\frac{3}{5}$ of B's age plus 44 years is equal to 5 times B's age; therefore $4\frac{2}{5}$ B's age is 44 years; or $\frac{22}{5}$ B's age=44 years; hence $\frac{1}{5}=2$ years, and his age is 10 years; and A's age is 20 years.

(20) As many, and half as many more, are $1\frac{1}{2}$ times as many dollars as he has; hence $1\frac{1}{2}$ times A's money plus $\$2\frac{1}{2}$ is $\$40$, or $1\frac{1}{2}$ times A's money$=\$37\frac{1}{2}$, or $\frac{3}{2}$ of A's money$=\$\frac{75}{2}$; hence $\frac{1}{2}$ is $\$\frac{25}{2}$, or A's money is $\$25$.

(21) $\frac{5}{2}$ of the number=100 less $2\frac{1}{2}=97\frac{1}{2}=\frac{195}{2}$; $\frac{1}{2}$ of the number$=\frac{39}{2}$ and the number=39.

(22) The whole minus $\frac{3}{8}$ is $\frac{5}{8}$. $\frac{4}{9}$ of $\frac{5}{8}=\frac{5}{18}$, and $\frac{5}{8}+\frac{5}{18}=\frac{45}{72}+\frac{20}{72}=\frac{65}{72}$; hence $\frac{65}{72}$ of the number at first is 65; therefore $\frac{1}{72}$ is 1 sheep, and $\frac{72}{72}$, or the whole flock, was 72 sheep.

(23) Since $\frac{5}{6}$ of the silver$+\$10=\frac{7}{4}$ of the gold, $\frac{1}{6}$ of the silver $+\$2=\frac{7}{20}$ of the gold, or the silver$+\$12=\frac{42}{20}$, or $\frac{21}{10}$ of the gold, or the silver$=\frac{21}{10}$ of the gold$-\$12$. But the gold and silver$=\$50$, and the gold$+\frac{21}{10}$ of the gold

—\$12=\$50, or $\frac{31}{10}$ of the gold=\$62. Hence $\frac{1}{10}$=\$2, and $\frac{10}{10}$, or the gold=\$20; therefore the silver is \$30.

(24) Since $\frac{1}{2}$ of A's—\$3=$\frac{1}{3}$ of B's+\$5, therefore $\frac{1}{2}$ of A's=$\frac{1}{3}$ of B's+\$8, or the whole of A's=$\frac{2}{3}$ of B's+\$16. But since both have \$56, therefore B's and $\frac{2}{3}$ of B's+\$16=\$56; or $\frac{5}{3}$ of B's+\$16 is \$56; therefore $\frac{5}{3}$ of B's is \$40. $\frac{1}{5}$ of \$40 is \$8, which is $\frac{1}{3}$ of B's, and $\frac{3}{3}$, that is, the whole of B's, is \$24. \$56—\$24=\$32, A's.

(25) Since $\frac{3}{7}$ of A's distance=$\frac{4}{5}$ of B's, therefore $\frac{1}{7}$ of A's distance=$\frac{1}{3}$ of $\frac{4}{5}$, or $\frac{4}{15}$ of B's, and $\frac{7}{7}$, or the whole of A's distance=$7\times\frac{4}{15}$, or $\frac{28}{15}$ of B's. $\frac{28}{15}$ of B's distance and B's distance, or $\frac{43}{15}$ of B's distance=86 miles. Hence $\frac{1}{15}$ of B's distance is $\frac{1}{43}$ of 86 miles, or 2 miles, and $\frac{15}{15}$ is 30 miles. A's distance is $\frac{28}{15}$, or 56 miles.

(26) Since $\frac{2}{3}$ of A's is equal to $\frac{4}{5}$ of B's, $\frac{1}{3}$ of A's is equal to $\frac{2}{5}$ of B's, and $\frac{3}{3}$, or the whole of A's, is equal to $\frac{6}{5}$ of B's. Hence their difference is $\frac{1}{5}$ of B's, and $\frac{3}{4}$ of $\frac{1}{5}$ of B's is \$15, that is, $\frac{3}{20}$ of B's is \$15; hence $\frac{1}{20}$ is $\frac{1}{3}$ of \$15, which is \$5, and $\frac{20}{20}$, or the whole of B's, is 20 times \$5, or \$100; and A's is $\frac{6}{5}$ of B's, which is \$120.

(27) Since the watch and chain are $\frac{9}{7}$ of the watch, therefore the chain is $\frac{2}{7}$ of the watch. 3 times $\frac{2}{7}$ are $\frac{6}{7}$, and twice the price of the watch plus $\frac{6}{7}$ of its price are $\frac{20}{7}$, which are \$100. Hence $\frac{1}{7}$ is $\frac{1}{20}$ of \$100, or \$5; and $\frac{7}{7}$, or the price of the watch is \$35, and the chain is $\frac{2}{7}$, or \$10.

(28) $\frac{7}{8}$ of 24 is 21; if 21 mi. are $\frac{3}{7}$, $\frac{1}{3}$ of 21, which is 7, is $\frac{1}{7}$ of the distance from B to C; hence $\frac{7}{7}$, or the distance, is 49 mi.; and from A to C is 49+24, or 73 mi.

(29) 4 horses for 2 months=8 horses for 1 month. Since 3 cows eat as much as 2 horses, 9 cows eat as much as 6 horses; hence 9 cows for 3 months=6 horses for 3 months=18 horses for 1 month. Again, since 10 sheep eat as much as 3 cows, and 3 cows as much as 2 horses, therefore 10 sheep eat as much as 2 horses, and 20 sheep as much as 4 horses; hence 20 sheep for 5 months=4 horses for 5 months=20 horses for 1 month. Hence, A has the same as 8 horses 1 month, B the same as 18 horses 1 month, and C the same as 20 horses 1 month, in all $8+18+20=46$ horses 1 month. 46 for \$92 is \$2 for 1 horse; hence A ought to pay $8\times\$2=\$16$; B $18\times\$2=\$36$; and C $20\times\$2=\$40$.

(30) Since the three eat 8 melons, each eats $2\frac{2}{3}$ melons: James eats $2\frac{2}{3}$ melons out of the 3 melons he furnished. Of the $2\frac{2}{3}$ melons eaten by Thomas, James furnished $\frac{1}{3}$, and John $2\frac{1}{3}$, that is, James furnished $\frac{1}{3}$, and John $\frac{7}{3}$, both together making $\frac{8}{3}$. Therefore, since James furnished $\frac{1}{8}$ of the $\frac{8}{3}$, and John $\frac{7}{8}$ of the $\frac{8}{3}$, James should have $\frac{1}{8}$ of the 8 cents, which is 1 cent; and John $\frac{7}{8}$ of the 8 cents, which are 7 cents.

(31) $\frac{2}{7}+\frac{1}{3}=\frac{6}{21}+\frac{7}{21}=\frac{13}{21}$; and $1-\frac{13}{21}=\frac{8}{21}$, C's part. $\frac{8}{21}-\frac{6}{21}=\frac{2}{21}$; hence $\frac{2}{21}$ is \$160, and $\frac{1}{21}$ is $\frac{1}{2}$ of $\$160=\$80$; $\frac{6}{21}=6$ times $\$80=\$480$, A's share; $\frac{7}{21}=7$ times $\$80=\$560$, B's share; and $\frac{8}{21}=8$ times $\$80=\$640$, C's share.

(32) Since A's age is $\frac{5}{3}$ of B's, twice A's age is $\frac{10}{3}$ of B's; hence $\frac{5}{3}$ B's age+B's age+$\frac{1}{2}$ B's age=$\frac{10}{3}$ B's age—2 years; or $\frac{19}{6}$ of B's age=$\frac{20}{6}$ of B's age—2 years; therefore $\frac{1}{6}$ of B's age is 2 years, and his age is 12 years; and $\frac{5}{3}$ of B's age, or A's age, is 20 years.

(33) A and B can do $\frac{1}{16}$ in 1 day, hence in 4 days they can do $\frac{4}{16}$ or $\frac{1}{4}$ of it, and $\frac{3}{4}$ remains. If B does $\frac{3}{4}$ in 36 days, he does $\frac{1}{3}$ of $\frac{3}{4}=\frac{1}{4}$, in 12 days, or $\frac{1}{12}$ of $\frac{1}{4}=\frac{1}{48}$, in 1 day, or the whole in 48 days. Since both can do $\frac{1}{16}$ in 1 day, and B can do $\frac{1}{48}$ in 1 day, A can do $\frac{1}{16}-\frac{1}{48}=\frac{1}{24}$ in 1 day, and the whole in 24 days.

(34) Since $\frac{2}{7}$ of B's and C's is \$12, $\frac{1}{7}$ is \$6, and $\frac{7}{7}$, or the sum of their shares, is \$42. Again, since $\frac{3}{8}$ of C's=$\frac{3}{10}$ of A's and B's, $\frac{1}{8}$ of C's=$\frac{1}{10}$ of A's and B's; or $\frac{10}{8}$, that is, $\frac{5}{4}$ of C's=A's and B's; but since A's is \$12, therefore $\frac{5}{4}$ of C's=\$12 and B's, or $\frac{5}{4}$ of C's less \$12=B's. Therefore C's and $\frac{5}{4}$ of C's less \$12=\$42, or $\frac{9}{4}$ of C's=\$54; hence $\frac{1}{9}$ of \$54, which is \$6, is $\frac{1}{4}$ of C's, and $\frac{4}{4}$, or the whole of C's, is \$24; and \$42 less \$24=\$18, B's share.

(35) To mow $\frac{1}{3}$ in 12 days would require 3 men; to do it in $\frac{1}{4}$ the time, 12 men; in $\frac{3}{4}$ the time, 4 men. *Ans.*

(36) During the first 6 months the second has $100 more than the first; to equalize it, then, the first must during the last 6 months have $100 more than the second, that is $300, and therefore must put in $200 *Ans.*

(37) After A had given B 2, they each had $\frac{1}{2}$ of 24=12 cts.; hence A had 12+2=14 cts., and B 12—2 or 10 cts.

(38) $\frac{3}{5}$ of 50=30. If 30 exceeds the age as much as 10 is less, 30—10=20 must be twice the excess, therefore the age is 30—10=20 years. *Ans.*

(39) The difference of 4 cts. in price makes 30 cts. difference in amount; hence there were $\frac{30}{4}=7\frac{1}{2}$ dozen; 12 cts.$\div 7\frac{1}{2}$ $=1\frac{3}{5}$ cts. loss per dozen; $6+1\frac{3}{5}=7\frac{3}{5}$ cts. cost per dozen.

(40) A difference of 3 cts. in price makes $1.50 cents difference in amount; hence there were $1.50÷3=50 lb. *Ans.*

(41) 12 boys=4 women=2 men. If 12 boys or 2 men can do the work in 6 days, it would require 6 times 2 men to do it in 1 day. *Ans.* 12 men.

(42) 2 lb. would be worth $3\frac{1}{2}+6\frac{1}{2}=10$ cts., or 5 cts. a lb.; $\frac{1}{2}$ lb. is worth $2\frac{1}{2}$ cts., and for $1 we must give 20 lb.

(43) If $\frac{2}{3}$ of the gain$=\frac{4}{15}$ of the selling price, $\frac{1}{3}$ will be $\frac{2}{15}$, and the gain $\frac{2}{5}$ of the selling price; hence the cost will be $1-\frac{2}{5}=\frac{3}{5}$ of the selling price. $3\frac{3}{4}$ yards at $4 cost $15 which is $\frac{3}{5}$ of $25. *Ans.*

(44) The first package cost 24—3=21 cts., and contained 21 ÷7=3 lb.; the second contained twice 3=6 lb., and cost 6×8=48 cts., and must be sold for 48+5 =53 cts.

SECTION 26.—PERCENTAGE.

NOTE.—The greater part of the questions in this section are of such easy solution, that it is deemed unnecessary to present either their answers, or the method of solving them.

LESSON II.

(2) *Ans.* 25. **(3)** *Ans.* 20. **(4)** *Ans.* 50. **(5)** *Ans.* 60. **(6)** *Ans.* 30.

(7) If he cleared $\frac{1}{5}$ he must have sold them for $\frac{6}{5}$ of the cost. If 6 cents was $\frac{6}{5}$, $\frac{1}{5}$ is 1 cent, and $\frac{5}{5}$, or the first cost, was 5 cents. Hence, in selling them for 8 cents, he cleared 3 cents on each; $\frac{3}{5}=60\ \%$.

(8) 6 yards for \$3 is $\$\frac{1}{2}$ a yard, and 5 yards for \$4 is $\$\frac{4}{5}$ per yard. $\frac{4}{5}-\frac{1}{2}=\frac{3}{10}$, $\frac{1}{2}$ is $\frac{5}{10}$, and $\frac{3}{10}$ is $\frac{3}{5}$ of $\frac{5}{10}$. $\frac{3}{5}$ is $\frac{60}{100}$, or 60 %.

(9) If he lost $\frac{1}{5}$, 8 cents are $\frac{4}{5}$ of the first cost, and the first cost was 10 cents. 3 for 25 cents is $8\frac{1}{3}$ cents a piece. 10 cents$-8\frac{1}{3}$ cents is $1\frac{2}{3}$ cents. $1\frac{2}{3}$ cents is $\frac{1}{6}$ of 10 cents; therefore by selling them at 3 for 25 cents, he would have lost $\frac{1}{6}$ of the cost, or $16\frac{2}{3}$ per cent.
Again: 2 for 25 cents is $12\frac{1}{2}$ cents a piece, a gain of $2\frac{1}{2}$ cents. $2\frac{1}{2}$ is $\frac{1}{4}$ of 10; hence by selling them at 2 for 25 cents, he would have gained $\frac{1}{4}$ of the first cost, or 25 %.

(10) 2 for 3 cents is $1\frac{1}{2}$ cents a piece; 3 for 2 cents is $\frac{2}{3}$ cent a piece. $1\frac{1}{2}-\frac{2}{3}$ is $\frac{5}{6}$ loss on each. $1\frac{1}{2}$ is $\frac{9}{6}$, therefore $\frac{5}{6}$ is $\frac{5}{9}$ of $1\frac{1}{2}$ cents, or the first cost; hence he lost $\frac{5}{9}$ of the first cost, or $55\frac{5}{9}\ \%$.

(12) *Ans.* 20. **(13)** *Ans.* 10. **(14)** *Ans.* 40. **(15)** *Ans.* 50.

(16) *Ans.* 12. **(17)** *Ans.* 3. **(18)** *Ans.* 25.

(19) $1-\frac{3}{5}=\frac{2}{5}=40\ \%$. At $\frac{7}{8}$ the loss is $\frac{1}{8}=12\frac{1}{2}\ \%$. At $\frac{9}{10}$ the loss is $\frac{1}{10}=10\ \%$. At $\frac{17}{20}$ the loss is $\frac{3}{20}=15\ \%$.

PROFIT AND LOSS.—LESSON III.

(1) $20\ \%=\frac{20}{100}=\frac{1}{5}$. *Ans.* $75\ \%=\frac{75}{100}=\frac{3}{4}$. *Ans.*

(2) $100\ \%=\frac{100}{100}=1$, that is, the cost and gain are equal. $150\ \%=\frac{150}{100}=1\frac{1}{2}$ or $\frac{3}{2}$; the gain is $1\frac{1}{2}$ times the cost.

(3) $25\ \%=\frac{25}{100}=\frac{1}{4}$ *Ans.* $35\ \%=\frac{35}{100}=\frac{7}{20}$. *Ans.*

(4) The whole, or $1=\frac{100}{100}$. *Ans.* 100 %. $\frac{3}{1}=\frac{300}{100}$ *Ans.* 300 %.

(6) If $\frac{2}{3}$ of the gain equals $\frac{6}{25}$ of the cost, $\frac{1}{3}$ equals $\frac{3}{25}$, and the whole gain equals $\frac{9}{25}=\frac{36}{100}=36\ \%$.

(7) If $\frac{5}{6}=\frac{15}{24}$, $\frac{1}{6}=\frac{3}{24}$, or $\frac{1}{8}$; and $\frac{6}{6}$, or the whole of the gain $=\frac{6}{8}$ or $\frac{3}{4}$ of the cost. $\frac{3}{4}=\frac{75}{100}=75\ \%$ *Ans.*

(8) 25 $\%=\frac{25}{100}=\frac{1}{4}$; hence the watch sold for $\frac{5}{4}$ of its cost; if \$10 is $\frac{5}{4}$, $\frac{4}{4}$, or the first cost, is \$8. When he sold it for \$12, he gained \$4, and $\frac{4}{8}=\frac{1}{2}=\frac{50}{100}=50$ %. *Ans.*

(9) $12\frac{1}{2}$ $\%=\frac{1}{8}$, when $\frac{1}{8}$ is lost, the muslin is sold for $\frac{7}{8}$ of its cost. Hence 7 cents is $\frac{7}{8}$ of the cost, and the first cost was 8 cents. By selling it at 6 cents per yard he lost 2 cents. $\frac{2}{8}=\frac{1}{4}=25$ %. *Ans.*

(10) $16\frac{2}{3}$ $\%=\frac{1}{6}$. If $\frac{1}{6}$ was lost the horse was sold for $\frac{5}{6}$ of his cost: if \$35 are $\frac{5}{6}$, $\frac{1}{6}$ is \$7, and $\frac{6}{6}$ are \$42, the cost. \$63—42=\$21, the gain; $\frac{21}{42}=\frac{1}{2}=\frac{50}{100}=50$ %. *Ans.*

(11) 20 $\%=\frac{20}{100}=\frac{1}{5}$; hence \$18 are $\frac{6}{5}$ of the first cost, or \$15. 10 $\%=\frac{1}{10}$, and $\frac{1}{10}$ of \$15 is $\$1\frac{1}{2}$. $15—1\frac{1}{2}=\$13\frac{1}{2}$, watch sold for. He lost $\$18—13\frac{1}{2}=\$4\frac{1}{2}$ *Ans.*

(12) 20 $\%=\frac{20}{100}=\frac{1}{5}$; hence \$60 are $\frac{6}{5}$ of the first cost, and $\frac{1}{6}$ of \$60 is \$10, which is $\frac{1}{5}$, or A's gain. B lost 20 %, or $\frac{1}{5}$ of what it cost him; $\frac{1}{5}$ of \$60 is \$12, B's loss. Hence B lost \$2 more than A gained.

(13) Since 25 % is $\frac{1}{4}$, he sold one watch for $\frac{5}{4}$ of its cost, and the other for $\frac{3}{4}$ of its cost; hence \$30 are $\frac{5}{4}$ of the first cost of the first watch, and $\frac{4}{4}$, or the first cost, is \$24, and he gained on the first watch \$6. If \$30 is $\frac{3}{4}$ of the first cost of the second watch, its first cost must have been \$40, and he lost \$10 by the sale; therefore, he lost \$10—\$6=\$4 *Ans.*

(14) 4 apples for 3 cents is $\frac{3}{4}$ of a cent each. 50 % is $\frac{1}{2}$; hence $\frac{3}{4}$ is $\frac{3}{2}$ of the first cost, and the first cost is $\frac{1}{2}$ cent. 5 apples for 4 cents is $\frac{4}{5}$ of a cent each; $\frac{4}{5}—\frac{1}{2}=\frac{3}{10}$, the gain on each apple. $\frac{1}{2}=\frac{5}{10}$, hence $\frac{3}{10}$ is $\frac{3}{5}$ of the first cost; $\frac{3}{5}=60$ %, the gain %.

(15) 5 lemons for 4 cents is $\frac{4}{5}$ of a cent each. 20 % is $\frac{1}{5}$, and he sold the lemons at $\frac{4}{5}$ of their cost. If $\frac{4}{5}$ of a cent is $\frac{4}{5}$ of the first cost, $\frac{1}{5}$ of a cent is $\frac{1}{5}$ of the cost, and the cost was 1 cent each. 6 lemons for 5 cents is $\frac{5}{6}$ of a cent each; $1—\frac{5}{6}=\frac{1}{6}=16\frac{2}{3}$ %, the % of loss.

(16) $\frac{2}{3}$ of 10 % of $60=\frac{2}{3}$ of $\frac{1}{10}$ of 60=4; 4 is $\frac{1}{2}$ of 8; 8 is $\frac{1}{5}$ of 40; $\frac{1}{5}=20$ % *Ans.*

(17) $\frac{1}{2}$ of $\frac{3}{5}$ of 50 % of 120=$\frac{3}{10}$ of $\frac{1}{2}$ of 120=18; 18 is 10 less than 28; 28 is 20 %, or $\frac{1}{5}$ of 5 times 28, which is 140. *Ans.*

LESSON IV.

(3) *Ans.* 60 cts. **(4)** *Ans.* $2. **(5)** *Ans.* $4.80

(6) *Ans.* $6. **(7)** *Ans.* $8. **(8)** *Ans.* $9.

(9) *Ans.* $8.40 **(10)** *Ans.* $9.

LESSON V.

(2) *Ans.* $1. **(3)** *Ans.* $2.80 **(4)** *Ans.* $2.40

(5) *Ans.* $4.50 **(6)** *Ans.* $3.25 **(7)** *Ans.* 5.16\frac{2}{3}$

(8) *Ans.* $3.53 **(9)** *Ans.* $4.80 **(10)** *Ans.* $19.20

(11) *Ans.* $8.45 **(12)** *Ans.* $4.24 **(13)** *Ans.* $3.20

(14) *Ans.* $6.75 **(15)** *Ans.* $3.80 **(16)** *Ans.* $28.

(17) *Ans.* $44. **(18)** *Ans.* $68.20 **(19)** *Ans.* $32.80

(20) *Ans.* $56.80 **(21)** *Ans.* $99.12

LESSON VI.

(2) *Ans.* $50. **(3)** *Ans.* $60. **(4)** *Ans.* $75.

(5) *Ans.* $140. **(6)** *Ans.* $240. **(7)** *Ans.* $350.

(8) *Ans.* $4000.

LESSON VII.

(2) *Ans.* $200. **(3)** *Ans.* $500. **(4)** *Ans.* $250.

(5) *Ans.* $300. **(6)** *Ans.* $25. **(7)** *Ans.* $750.

LESSON VIII.

(2) *Ans.* 4 years. **(3)** *Ans.* 2$\frac{1}{2}$ years. **(4)** *Ans.* 2$\frac{2}{3}$ years.

(5) *Ans.* 3$\frac{3}{7}$ years. **(6)** *Ans.* 6$\frac{2}{3}$ years.

(8) *Ans.* 50, 33$\frac{1}{3}$, 20, 16$\frac{2}{3}$, 14$\frac{2}{7}$, 12$\frac{1}{2}$, 10, and 8$\frac{1}{3}$ years respectively.

(9) *Ans.* 40 years. **(10)** *Ans.* 25 years, and 20 years.

LESSON IX.

(2) *Ans.* 8. **(3)** *Ans.* 5. **(4)** *Ans.* 7. **(5)** *Ans.* 8.
(6) *Ans.* 7. **(7)** *Ans.* 5. **(8)** *Ans.* 6. **(10)** *Ans.* $8\frac{1}{3}$.
(11) *Ans.* 10. **(12)** *Ans.* $12\frac{1}{2}$, 20, 25, and 50, respectively.

LESSON X.

(2) *Ans.* $400. **(3)** *Ans.* $25. **(4)** *Ans.* $500. **(5)** *Ans.* $300.

(6) The amount of $1 for 4 years, at 6 %, is $1.24; and the required principal will be as many dollars as $1.24 is contained times in $496 : $496÷$1.24=400. *Ans.* $400.

(7) $1, at 5 %, for 4 years, will amount to $1.20; hence the present worth of each $1.20 of the given sum, is $1; therefore, the present worth of $24 will be as many times $1, as $1.20 is contained times in $24 : $24÷$1.20=20. *Ans.* $20.

(8) The amount of $1 for 5 years, at 6 %, is $1.30, and the present worth of each $1.30 of the amount is $1 ; therefore, the present worth of $65 will be as many times $1, as $1.30 is contained times in $65: $65÷$1.30=50; hence the present worth is $50. By subtracting the present worth from the debt, we obtain the discount: $65—$50=$15.

(9) The amount of $1 for 5 years, at 5 %, is $1.25; hence the present worth of each $1.25 of the given sum, is $1. And the present worth of $55, will be as many dollars as $1.25 is contained times in $55 : $55÷$1.25 =44; therefore, $44 is the present worth. $55—$44= $11, the discount.

(10) 6 years 8 months=$6\frac{2}{3}$ yr. If the interest of $1 for 1 year is 6 cents, for $6\frac{2}{3}$ years it will be 40 cts.=$\$\frac{2}{5}$; hence the amount of $1 for $6\frac{2}{3}$ years, at 6 %, is $\$1\frac{2}{5}=\$\frac{7}{5}$. Therefore the present worth of $77 will be as many dollars as $\frac{7}{5}$ is contained times in 77, which is 55. Hence the present worth is $55, and the gain by paying it now $77—$55=$22.

LESSON XI.

MISCELLANEOUS EXAMPLES IN INTEREST.

(1) $6 \times 4\frac{1}{6} = 25$; $\frac{25}{100} = \frac{1}{4}$. *Ans.*

(2) $5 \times 5 = 25$; $\frac{25}{100} = \frac{1}{4}$. $1 + \frac{1}{4} = \frac{5}{4}$, and $\frac{1}{4}$ is $\frac{1}{5}$ of $\frac{5}{4}$. *Ans.* $\frac{1}{5}$

(3) $\frac{1}{2}$ of $\frac{1}{5} = \frac{1}{10}$; hence the interest of 1 year is $\frac{1}{10}$ of the principal; $\frac{1}{10} = \frac{10}{100}$ *Ans.* 10 %.

(4) 2 years 6 months $= 2\frac{1}{2} = \frac{5}{2}$ years. $\frac{1}{5}$ of $\frac{1}{4} = \frac{1}{20}$, and 2 times $\frac{1}{20} = \frac{1}{10}$, the part the interest is of the principal for 1 year; $\frac{1}{10} = \frac{10}{100} = 10$ %. *Ans.*

(5) 10 % $= \frac{10}{100} = \frac{1}{10}$; hence for 1 year the interest is $\frac{1}{10}$ of the principal, and at the same rate %, the time for which the interest is $\frac{3}{5}$ of the principal, will be as many years as $\frac{1}{10}$ is contained times in $\frac{3}{5}$. $\frac{1}{10}$ in $\frac{3}{5}$ is contained 6 times. *Ans.* 6 years.

(6) If 3 times the yearly interest is equal to $\frac{9}{25}$ of the principal, once the yearly interest is equal to $\frac{3}{25}$ of the principal. $\frac{3}{25} = \frac{12}{100} = 12$ %. *Ans.*

(7) $\frac{1}{5}$ of the interest for 2 years is the same as $\frac{2}{5}$ of the interest for 1 year.

If $\frac{2}{5}$ of the interest for 1 year equals $\frac{4}{25}$ of the principal, $\frac{1}{5}$ of the interest for 1 year equals $\frac{2}{25}$ of the principal, and $\frac{5}{5}$, or the interest for 1 year, equals $\frac{10}{25}$ of the principal. $\frac{10}{25} = \frac{40}{100} = 40$ %. *Ans.*

(8) $\frac{5}{8}$ for 3 years is the same as $\frac{15}{8}$ for 1 year. If $\frac{15}{8}$ of the interest for 1 year equals $\frac{9}{80}$ of the principal, $\frac{1}{8}$ equals $\frac{1}{15}$ of $\frac{9}{80} = \frac{3}{400}$, and $\frac{8}{8}$ equals $\frac{3}{50}$, or $\frac{6}{100}$ of the principal. $\frac{6}{100}$ equals 6 %. *Ans.*

(9) 1 year 4 months $=$ 16 months, which is twice 8 months; hence the interest for 1 year 4 months will be $\frac{2}{25}$ of the principal; $\frac{2}{25}$ of \$200 $=$ \$16. *Ans.*

(10) 1 year 4 mon. are $1\frac{1}{3}$, or $\frac{4}{3}$ yr. If the interest of $\frac{4}{3}$ yr. is $\frac{3}{25}$ of the principal, for $\frac{1}{3}$ year it is $\frac{1}{4}$ of $\frac{3}{25} = \frac{3}{100}$, and for 1 year it is $\frac{9}{100}$ of the principal; that is, the rate is 9 %. Hence for 1 year the interest is $9; for 1 month the interest is $\$\frac{3}{4}$, and for 8 mon. $6; for 18 days the interest is $\frac{18}{30}$, or $\frac{3}{5}$ of $\$\frac{3}{4}=\$\frac{9}{20}$, or 45 cents. Therefore, the whole interest is $\$9+\$6\frac{9}{20}=\$15\frac{9}{20}$, or $15.45

(11) 4 years at 10 % is the same as 1 year at 40 %, and 1 year at 40 % is the same as 8 years at 5 %. *Ans.* 8 years.

(12) $3\frac{1}{3}$ years are $\frac{10}{3}$ years. $40 for $\frac{10}{3}$ years is $4 for $\frac{1}{3}$ of a year, or $12 a year. 5 % is $\frac{1}{20}$, hence $12 are $\frac{1}{20}$ of the principal, which must therefore be 12 times $20= $240. Hence B's money, and twice B's money, or 3 times B's money is $240; therefore B's money is $80, and A's twice $80, or $160.

(13) $1\frac{2}{5}$ years are $\frac{7}{5}$ years. $49 for $\frac{7}{5}$ years are $7 for $\frac{1}{5}$ year, or $35 a year. 7 % is $\frac{7}{100}$, hence $35 are $\frac{7}{100}$ of the principal, and $5 are $\frac{1}{100}$, and $\frac{100}{100}$ are $500, the sum of A's and B's money. Since 2 times A's equals 3 times B's, 1 time A's equals $1\frac{1}{2}$ B's; therefore B's money and $1\frac{1}{2}$ times B's money, that is, $\frac{5}{2}$ are equal to 500; hence $\frac{1}{2}$ B's money is $\frac{1}{5}$ of $500, or $100, therefore B's money is $200, and A's $300.

(14) 2 yrs. 3 mos.$=2\frac{1}{4}=\frac{9}{4}$ yrs. $18 for $\frac{9}{4}$ yrs.$=\$2$ for $\frac{1}{4}$ year, or $8 for 1 year. 4 % is $\frac{4}{100}=\frac{1}{25}$; hence $\$8=\frac{1}{25}$ of the principal, and $\frac{25}{25}$, or the whole principal, is $200, which is $\frac{3}{4}$ of A's, and $\frac{1}{2}$ of B's money. Since $\frac{1}{2}$ A's $=\frac{2}{3}$ B's money, $\frac{1}{4}$ A's$=\frac{1}{3}$ B's, and $\frac{3}{4}$ A's=B's money; hence B's money and $\frac{1}{2}$ B's, or $\frac{3}{2}$ B's=$200; $\frac{1}{3}$ of $200 $=66\frac{2}{3}$, which is $\frac{1}{2}$ B's, and $\$133\frac{1}{3}=$B's money. Since $\frac{1}{2}$ A's$=\frac{2}{3}$ B's money, A's money$=\frac{4}{3}$ B's, and $\frac{4}{3}$ of $\$133\frac{1}{3}$ $=\$177\frac{7}{9}$, A's money.

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